The Origins and Propagation of Animal Spirits Shocks*

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Abstract

This paper attributes the origin of aggregate fluctuations to the stochastic synchronization of firms' investment spikes, which we call investment avalanches. In a monopolistic competition model with idiosyncratic productivity shocks, firms' decisions on lumpy investments exhibit complementarity in equilibrium due to aggregate demand externality. A firm's investment can cause other firms' investments, which in turn cause other firms, and so forth. The magnitude of this investment avalanche depends on the distribution of firms' productivity and capital states. We illustrate that variations in avalanche size generate significant fluctuations in aggregate investment demand in a model lacking exogenous aggregate shocks. We demonstrate that a New Keynesian business cycle model with investment demand shocks naturally yields procyclical inflation. Real wage rigidity and diminishing returns to labor, stemming from

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the presence of capital, are crucial in our propagation mechanism: a positive investment demand shock increases labor demand, decreases the marginal product of labor, and raises the marginal cost of final goods production. The model implies that an optimal monetary policy accommodates some inflation resulting from the investment demand shock. Conversely, the optimal policy induces disinflation when a positive productivity shock impacts investments.

1 Introduction

A defining feature of business cycles is the comovement of consumption, investment, and inflation with output. The cause of the high unconditional correlation remains an open question, and various tentative explanations can be identified in the extensive literature reviewed below. First, a positive supply shock can simultaneously increase consumption and investment, but it appears deflationary. Second, news about a supply shock can generate positive comovement between consumption and inflation, but it decreases inflation. Third, a positive investment-specific technological (IST) shock can create positive comovement between investment and inflation, but consumption declines. Finally, a shock to the discount factor can raise consumption and inflation while decreasing investment. This comovement puzzle, already discussed in Barro and King (1984), is thus difficult to resolve with a single shock. An interaction of two shocks occurring simultaneously can engender positive comovements among three variables. Still, one must acknowledge that it merely shifts the puzzle to the systematic correlation between the two shocks in the data.

In this paper, we reproduce a positive comovement of the three variables with a new type of investment shocks, labeled as an "animal spirits" shock for reasons clarified below, which is a microfounded coordination failure. While an IST shock is essentially a technology shock that requires an efficient economy to respond to, our investment shock is inefficient. Consequently, the animal spirit shock generates excess volatility in investment and consumption, which may call for a stabilization policy. Thus, the model of investment fluctuations is not only essential for understanding the data, but it also has important policy implications.

We provide a microfoundation for the investment demand shock without relying on exogenous aggregate shocks. We consider a monopolistic competition model where firms' investment decisions are complementary in equilibrium due to an aggregate demand pecuniary externality: an increase in a firm's capital boosts demand for goods and subsequently leads to investments by other firms. We assume that capital is indivisible up to a lumpiness parameter. Thus, the number of firms investing in a period, or the extensive margin of capital adjustments, determines aggregate investment demand. Even when the economy consists of a finite large number n of firms, the aggregate investment demand deviates from the steady-state level due to finiteness. This deviation is amplified by the complementarity in investment decisions. We will show that this amplification effect, termed an investment avalanche, leads to a qualitatively different distribution of the number of investing firms than the central limit theorem predicts and is quantitatively significant even when n is very large.

This mechanism generates comovements in consumption, investment, and inflation within a New Keynesian model that features standard elements such as wage rigidity, capital, and constant returns to scale. When a positive investment demand shock affects an economy, an increase in capital in the next period encourages households to consume more in the current and following periods through wealth effects. An uptick in investment demand also tightens the final goods market and expands labor demand. With sticky real wages and diminishing returns to labor due to the presence of capital, an increase in labor input reduces the marginal product of labor. Therefore, intermediate producers pass on the heightened production costs to final goods producers who encounter price stickiness, resulting in a rise in both output and the price level.

Our model does not employ exogenous aggregate shocks, aligning with the literature on the origin of aggregate fluctuations. Gabaix (2011) attributed the origin to idiosyncratic productive shocks affecting large firms, represented in the tail of a power-law distribution of firm size. Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) identified the aggregate consequences of key sectors when a power-law distribution characterizes sectoral influence through input-output networks. These papers provided microfoundations for aggregate technological shocks. Our paper differs from theirs in two key aspects. First, we focus on aggregate demand shocks rather than productivity shocks. Second, our mechanism does not involve the granular effect. We assume firms have a homogeneous size, abstracting from the aggregate consequences of large or critically influential firms. In our model, a firm's lumpy investment induces another, triggering a chain reaction. The extent of this chain reaction is stochastic, as it is determined by firms' idiosyncratic productivities and initial capital levels. The resulting multiplier effect follows a power-law distribution under constant returns to scale technology, producing aggregate fluctuations from idiosyncratic shocks.

Our model is fully compatible with conventional models that incorporate various exogenous shocks, even though it does not require them to generate business fluctuations. Unlike the responses to exogenous shocks affecting fundamentals, the aggregate investment fluctuations in our model are inefficient for representative households. This inefficiency arises from a coordination failure among households as investors. If households could collectively decide on aggregate investments, they might prefer a smoother path for investment and consumption. However, when the intermediate producer at the extensive margin of aggregate investments makes its lumpy investment decisions in a decentralized manner, the complementarity effect dominates its choice. In this regard, the underlying economic condition for our animal spirits shocks is imperfections in financial markets, where each investing firm perceives expected discounted marginal returns to investments differently than when representative households determine aggregate investments.

The volatile investment demand in our model seems to stem from animal spirits, as it lacks a corresponding identifiable aggregate exogenous shock, such as shocks to technology, preference, or information. The aggregate investment in the model fluctuates based on subtle changes in the firm-level capital and productivity profile. The aggregate investment data produced by our model will appear to be driven by an exogenous shock that seemingly emerges from nowhere from an aggregate perspective. This paper aligns with Angeletos and La'O (2013) in seeking the origin of animal spirits shocks not in investors' psychology, but in coordination failures in imperfect markets. While our model complements their approach, it differs in that the aggregate fluctuations arise from current and past idiosyncratic productivity shocks without correlated exogenous shocks.

Related literature This paper builds on the extensive literature regarding the origin of business cycles and the role of investment demand. Our model closely aligns with the business cycle literature concerning investment shocks. Fisher (2006) found empirical evidence for technological shocks in the investment goods sector and explored its implications in business cycles. Justiniano, Primiceri, and Tambalotti (2010) established an important role played by an investment technological shock in an estimated business cycle model and interpreted it as a shock originating from the financial sector that transforms investment into effective capital. Christiano, Motto, and Rostagno (2014) extended this view and emphasized the role of risk shocks, as discussed in Bloom (2009), within an estimated New Keynesian model. Beaudry and Portier (2014) investigated how news about future productivity affects current capital formations.

While these studies connect the overall fluctuation to an external shock in aggregate production technology, shared information, or the financial environment that transforms investment expenditures into capital increases, we focus on the interactions between the simultaneous investment decisions of multiple firms. While an investment technological shock increases capital without consuming resources, an investment demand shock in our model utilizes contemporaneous resources, leading to a different response in inflation. In this regard, our model relates to a time preference shock but differs in two main ways. First, a time preference shock directly creates a trade-off between consumption and investment, whereas our model features distinct decision-making processes for households and firms. Second, investment fluctuations stemming from time preference shocks reflect the economy's efficient responses. In contrast, investment shocks in our model are inefficient for households because they indicate a coordination failure among investors who cannot control aggregate investment levels.

Our model follows the tradition of sectoral business cycles of Long and Plosser (1983), which feature realistic technological shocks at the sectoral level. However, the aggregate fluctuations generated from idiosyncratic shocks suffer a diversification effect, whereby the volatility of aggregate fluctuations decreases quickly as the number of sectors increases, as discussed by Dupor (1999) and Horvath (2000). Incorporating non-linear behavior at the micro level, such as in (S,s) models, may open the possibility of circumventing the diversification effect. Nevertheless, previous research has shown that the non-linear behaviors produce weak aggregate fluctuations (Caplin and Spulber, 1987; Caballero and Engel, 1991; Thomas, 2002; Khan and Thomas, 2003, 2008).

In contrast, this paper illustrates that the (S,s) behavior can cause aggregate fluctuations. In our model, firms' investments exhibit strategic complementarity in equilibrium as defined in Caballero and Engel (1993). In an environment with a finite number of firms rather than a continuum of firms, we show that the complementarity in lumpy investments can generate sizable aggregate fluctuations. Our effort relates to recent studies on the role of interest elasticity of investment demand in the diversification effect operating in an (S,s) economy (Auclert, Rognlie, and Straub, 2020; Koby and Wolf, 2020; Winberry, 2021; Zwick and Mahon, 2017).

Our model embodies the spirit of endogenous business cycle studies, such as sunspot models by Galí (1994) and Wang and Wen (2008), among others. It incorporates micro-level independent shocks but excludes exogenous aggregate shocks. In this respect, our model can generate aggregate fluctuations endogenously. Unlike the sunspot and indeterminacy models, it produces a locally unique equilibrium rather than a continuum of equilibria. We select a locally unique equilibrium that is closest to an equilibrium of a "smooth" economy, where the lumpiness of investments is irrelevant to aggregates. The local uniqueness implies that the equilibrium of a smooth economy cannot be generically realized as an equilibrium of an (S,s) economy with finite firms. We then demonstrate that the equilibrium diverges from its smooth counterpart in a quantitatively significant way.

Many authors have studied the departure from the diversification effect, including Brock and Durlauf (2001), Gabaix (2011), and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). The underlying analytics of our paper's fluctuation mechanism are based on power-law distributions. In contrast to the granular hypothesis of Gabaix (2011), our model incorporates a stochastic synchronization called avalanches. The so-called self-organized criticality model, which generates avalanches, has been examined by Scheinkman and Woodford (1994), Nirei (2006), and Nirei, Stachurski, and Watanabe (2020). The investment avalanche utilized in this paper was proposed by Nirei (2015) in a context of real business cycles, and a rigorous analysis of avalanches within an (S,s) model has been recently presented by Nirei and Scheinkman (2024).

Lumpy investments The key feature we incorporate into an otherwise standard business cycle model is lumpy investments. Authors such as Cooper and Haltiwanger (1996) and Gourio and Kashyap (2007) have emphasized the crucial role that lumpy investments play in business cycles. We briefly summarize this point using a Japanese business survey (BSJBSA).[1] We define a lumpy investment as an incident where a firm's annual gross investment exceeds 20% of its outstanding capital. In our dataset, lumpy investments occur at a rate of 13.5% with a standard deviation of 21.6%. The ratio of aggregate lumpy investment to total aggregate investment averages 33% during the sample period. We observe a clear comovement between the growth rates of lumpy and total investment, with the correlation coefficient exceeding 95%. This confirms the salient pattern that lumpy investments drive aggregate investments.

Impulse responses of inflation The empirical target of this paper is the impulse response functions of output, consumption, and inflation to an exogenous investment shock. Figure 1 presents an empirical estimate of the impulse response functions. First, we estimate an exogenous aggregate investment shock by orthogonalizing the real investment growth rates with respect to the predicted total factor productivity (TFP) series up to the 4th lead, as well as the past TFP and investment series up to the 4th lag. We utilize an updated estimate of a utilization-adjusted TFP series (for non-equipment output) by Fernald (2014). Then, we estimate a structural VAR model with Cholesky identification in the order of the exogenous investment shock, the utilization-adjusted TFP shock, growth rates of real GDP, real consumption, and CPI.[2] Expectedly, the exogenous investment shock displays a non-significant effect on the current and future TFP shocks.

[1] The Ministry of Economy, Trade and Industry conducts the Basic Survey of Japanese Business Structure and Activities. It covers firms with 50 or more employees and 30 million yen or more in capital. The response rate in 2022 was 90.2%. In our database, the survey consists of an unbalanced panel of 31197 firms from 2007 to 2021. See Nirei (2024) for details.

[2] We use the NIPA quarterly data for 1947Q2-2024Q4 on real GDP, gross private domestic investment, hours worked for all workers in the nonfarm business sector, and the Consumer Price Index for all urban consumers: all items in U.S. city average. All growth rates are annualized.

Figure 1 illustrates the estimated impulse response functions for a one-percentage-point exogenous investment shock and a TFP shock. We note that the investment shock leads to an increase in GDP, consumption, and inflation, while the TFP shock does not induce inflation. Our goal is to develop a theory that elucidates this pattern.

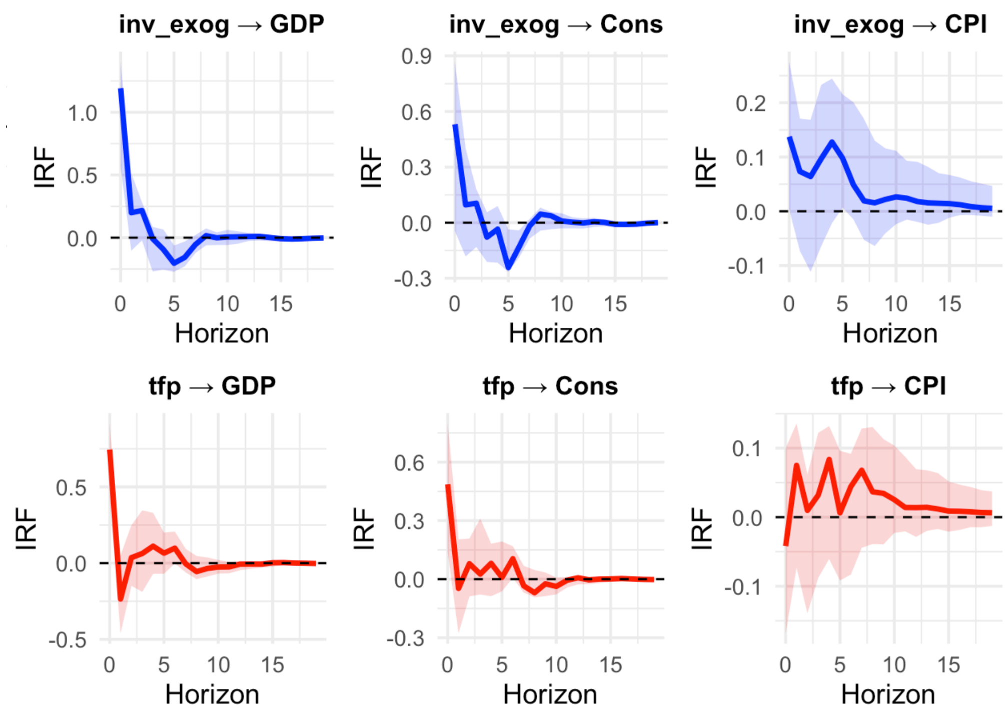

Figure 1: Impulse response functions of the growth rates of GDP, consumption, and CPI for a one percentage point shock on an exogenous investment shock (top) and a TFP shock (bottom). The horizontal axis represents quarters, and the vertical axis represents percentages. Shadowed areas indicate 95% confidence intervals.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 examines the impulse-response analysis of an aggregate investment demand shock within the model. Section 4 explores the mechanism behind an investment avalanche that causes the aggregate investment demand shock. Section 5 concludes. The appendix includes proofs and extensions.

2 Model

The model consists of representative final good producers, wholesalers, intermediate good producers, representative households, and the central bank. Households consume final goods, supply labor, own firms, and have access to risk-free assets. Intermediate producers monopolistically produce differentiated goods using capital and labor. Wholesalers aggregate intermediate goods, incur price adjustment costs, and sell them to final goods producers. Final goods are used for consumption and investment. Intermediate producers own capital and make investment decisions. Our model features indivisible capital, where intermediate producers can choose the capital level only discretely. In the remainder of this section, we present parametric specifications and define an equilibrium. Detailed derivations are provided in the online appendix.

2.1 Production

Final goods Final good Y_t is produced using a CES aggregator function,

$$Y_t = \left(\int y_{it}^{\frac{\epsilon_c - 1}{\epsilon_c}} di \right)^{\frac{\epsilon_c}{\epsilon_c - 1}},$$

where $\epsilon_c > 1$, and is competitively supplied. Denoting the price of y_{it} as p_{it}, the competitive price of Y_t is given by $P_t = \left(\int p_{it}^{1-\epsilon_c} di \right)^{\frac{1}{1-\epsilon_c}}$, and the minimized cost is $\int p_{it} y_{it} di = P_t Y_t$. The derived demand for y_{it} is expressed as $y_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\epsilon_c} Y_t$.

Wholesaler $i \in [0, 1]$ is a monopolistic supplier of y_i with quadratic price adjustment costs à la Rotemberg. Wholesalers are symmetric. The production function of wholesaler i is $y_{it} = Y_{it}^m$, where Y_{it}^m is a CES aggregate of intermediate goods with an elasticity of substitution $\eta > 1$,[3]

$$Y_{it}^m = \left(\sum_{j=1}^{n} (y_{ijt}^m)^{(\eta-1)/\eta} / n \right)^{\eta/(\eta-1)}.$$

The linear production function and symmetry imply that $Y_t = Y_t^m = Y_{it}^m$ for all i.

[3] When the number of intermediate goods is taken to infinity, $n \to \infty$, the CES aggregate converges to its continuum counterpart, $(\int (y_{ijt}^m)^{(\eta-1)/\eta} dj)^{\eta/(\eta-1)}$, in an economy with measure one intermediate firms $j \in [0, 1]$ (see Nirei and Scheinkman, 2024).

The real value of wholesaler i is the expected discounted sum of its future dividends, $\max \mathbb{E}\left[\sum_t \Lambda_t \Omega_{it}\right]$, where Λ_t denotes the stochastic discount factor and

$$\Omega_{it} := \frac{p_{it} Y^m_{it} - (1-\tau^Y_t) \sum_{j=1}^n p^m_{jt} y^m_{ijt}/n}{P_t} - \frac{\psi_P}{2}\left(\frac{p_{it}}{p_{i,t-1}} - 1\right)^2 Y_t - t^Y_t,$$

subject to production and demand functions, where t^Y_t represents the standard lump-sum tax financing the subsidy τ^Y_t, aiming at undoing steady state distortions.

A wholesaler's minimized unit cost of production is symmetric across wholesalers and equal to $P^m_t = \left(\sum_{j=1}^n (p^m_{jt})^{1-\eta}/n\right)^{1/(1-\eta)}$. The derived demand of wholesaler i for intermediate good j is $y^m_{ijt} = (p^m_{jt}/P^m_t)^{-\eta} Y^m_{it}$. By aggregating across symmetric i, we obtain the total demand for j as $y^m_{jt} = (p^m_{jt}/P^m_t)^{-\eta} Y_t$.

The relative price of the intermediate composite is denoted by $m_t := P^m_t/P_t$. The markup charged by the wholesale sector is $1/m_t$. The inflation rate is $\pi_t := P_t/P_{t-1} - 1$. Using these notations, the equilibrium aggregate profits of the wholesale are expressed as $\Omega_t := \int \Omega_{it} di = (1 - m_t - (\psi_P/2)\pi_t^2) Y_t$.

Wholesalers maximize their value by choosing their prices. We derive the following New Keynesian Phillips curve after the usual derivations and by setting $\tau^Y_t = 1/\epsilon_c$:

$$\pi_t(1+\pi_t) = \frac{\epsilon_c - 1}{\psi_P}(m_t - 1) + \mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{Y_{t+1}}{Y_t}\pi_{t+1}(1+\pi_{t+1})\right]. \tag{1}$$

The stationary inflation rate is zero, and the steady-state price of intermediate goods is $m = 1$. Symmetry across wholesaler i implies that $y^m_{jt} = y^m_{ijt}$ and $Y^m_t := \int Y^m_{jt} dj = Y^m_{it}$ for any i.

Intermediate goods Intermediate goods are differentiated, with each intermediate good $j = 1, 2, \ldots, n$ supplied by a monopolist firm j that uses the production function $y^m_{jt} = a_{jt} k^\alpha_{jt} l^{1-\alpha}_{jt}$. In Appendix A, we generalize this production function to the case of decreasing returns to scale.

Our model features a lumpy investment behavior. For simplicity, we assume that capital is indivisible up to a lumpiness parameter λ. Specifically, we consider that an intermediate producer is subject to a discrete capital constraint $k_{jt} \in \{k_{j,t-1}(1-\delta)\lambda^s\}_{s=0,\pm 1}$, where the lumpiness parameter satisfies $(1-\delta)\lambda > 1$. While it is possible to endogenize λ by considering a non-convex cost of adjustments,[4] we maintain the indivisibility assumption for the sake of expositional simplicity.

Intermediate firm j encounters demand from wholesalers as $y^m_{jt} = \left(p^m_{jt}/P^m_t\right)^{-\eta} Y_t$. Firm j's investment is represented by $x_{j,t} := k_{j,t+1} - (1-\delta)k_{j,t}$. The real value of intermediate producer j is expressed as $\max \mathbb{E}\left[\sum_t \Lambda_t(\mu_{jt} - x_{jt})\right]$, where μ_{jt} denotes real operating surplus as $\mu_{jt} := \frac{p^m_{jt}}{P_t} y^m_{jt} - w_t l_{jt}$ and w_t represents real wages. In response to the demand function derived above, firm j determines labor demand as $l_{jt} = (1-1/\eta)(1-\alpha)(m_t/w_t)(y^m_{jt})^{1-1/\eta} Y_t^{1/\eta}$. Aggregating across all firms j, we obtain the aggregate goods supply and labor demand functions:

$$Y_t = ((1-1/\eta)(1-\alpha)m_t/w_t)^{\frac{1-\alpha}{\alpha}} K_t, \tag{2}$$

$$L_t = \sum_{j=1}^{n} l_{jt}/n = (1-1/\eta)(1-\alpha)(m_t/w_t)Y_t, \tag{3}$$

where

$$K_t := \left(\sum_{j=1}^{n} (a_{jt}^{1/\alpha} k_{jt})^{\rho}/n\right)^{1/\rho} \quad \text{and} \quad \rho := \frac{(1-1/\eta)\alpha}{1-(1-1/\eta)(1-\alpha)}.$$

Since $\eta > 1$ and $0 < \alpha < 1$, we obtain $0 < \rho < 1$.

At optimal factor demands, the operating surplus is expressed as a function of productivity and capital $\mu_t(a_{jt}, k_{jt})$ as

$$\mu_t(a_{jt}, k_{jt}) = \kappa (a_{jt}^{1/\alpha} k_{jt})^{\rho} m_t^{1/\alpha} w_t^{1-1/\alpha} K_t^{1-\rho} \tag{4}$$

where $\kappa := (1-(1-1/\eta)(1-\alpha))\left((1-1/\eta)(1-\alpha)\right)^{(1-\alpha)/\alpha}$.

The operating surplus function μ_t in (4) is strictly concave in k_{jt} since $0 < \rho < 1$. Aggregating (4) yields an expression for total operating surplus:

$$\sum_{j=1}^{n} \mu_t(a_{jt}, k_{jt})/n = \kappa m_t^{1/\alpha} w_t^{1-1/\alpha} K_t. \tag{5}$$

2.2 Lumpy investment

An intermediate producer's capital choice is subject to an indivisibility constraint, which plays a central role in our analysis. Because the capital choice of an intermediate firm is

[4]For example, see Nirei and Scheinkman (2024) in the context of menu-cost pricing.

limited by a discrete set, the firm optimally chooses a threshold of capital for an investment spike that increases the firm's capital by a factor of λ. The firm can either choose an investment spike in t and no spike in $t+1$, or no spike in t and a spike in $t+1$, which leaves the firm at the same level of capital in $t+2$. At the threshold, firm j is indifferent between the two alternative plans. Therefore, the optimal threshold of the investment spike, $k^* = k_{j,t+1}$, satisfies the following indifference condition:

$$\mathbb{E}_t \Lambda_{t+1}(\mu_{t+1}(a, k^*) + (1-\delta)k^*) - \Lambda_t k^* = \mathbb{E}_t \Lambda_{t+1}(\mu_{t+1}(a, \lambda k^*) + (1-\delta)\lambda k^*) - \Lambda_t \lambda k^*.$$

Solving for k^*, we obtain the optimal threshold for an investment spike:

$$k^*_{j,t+1} = a^{\eta-1}_{j,t+1} \Phi_t K_{t+1}, \tag{6}$$

where Φ_t summarizes the expected factor prices

$$\Phi_t := \left(\kappa \frac{\lambda^\rho - 1}{\lambda - 1} \mathbb{E}_t \left[\Lambda_{t+1} m^{1/\alpha}_{t+1} w^{1-1/\alpha}_{t+1} \right] \mathbb{E}_t \left[\Lambda_t - \Lambda_{t+1}(1-\delta) \right]^{-1} \right)^{\frac{1}{1-\rho}}.$$

We note that the threshold policy is linear in aggregate capital.

The optimal inaction region for firm j in period t is $k_{j,t} \in [k^*_{j,t+1}/(1-\delta), \lambda k^*_{j,t+1}/(1-\delta))$. The support of a stationary distribution is $k_{j,t} \in [k^*_{j,t}, \lambda k^*_{j,t})$. We define a state variable s_{jt} for firm j that indicates the distance of log capital from the threshold level as:

$$s_{jt} := \frac{\log k_{jt} - \log k^*_{jt}}{\log \lambda}. \tag{7}$$

Let F_t represent a joint distribution of (a_{jt}, s_{jt}) and F be the stationary distribution. Aggregate capital is expressed using F_t as $K_t = \left(\sum_{j=1}^n (a^{1/\alpha}_{jt} k_{jt})^\rho / n \right)^{1/\rho} = \mathbb{E}^{F_t} \left[(a^{1/\alpha}_{jt} \lambda^{s_{jt}} k^*_{jt})^\rho \right]^{1/\rho}$. Substituting $k^*_{j,t+1}$ into this expression for K_{t+1} results in an equilibrium condition for expected factor prices, Φ_t:

$$1 = \mathbb{E}^{F_{t+1}} \left[a^{\eta-1} \lambda^{\rho s} \right]^{1/\rho} \Phi_t. \tag{8}$$

2.3 Households

Representative households' utility function is given by $\mathbb{E}_0 \left[\sum_{t=0}^\infty \beta^t (u(C_t) - v(N_t)) \right]$, where u is strictly concave and v is convex. Households own monopolistic firms and earn profits from wholesale Ω_t, and intermediate producers, $\sum_{j=1}^{n}(\mu(a_{jt}, k_{jt}) - x_{jt})$. They have access to a risk-free nominal asset, A_t, at a risk-free nominal rate of i_t. Households' budget constraint in nominal terms is expressed as

$$A_t + P_t C_t = (1 + i_{t-1})A_{t-1} + P_t \left(w_t N_t + \Omega_t + \sum_{j=1}^{n} (\mu(a_{jt}, k_{jt}) - x_{jt})/n \right).$$

The choice of A_{t+1} leads to the Euler equation:

$$u'(C_t) = \beta \mathbb{E}_t \left[\frac{1 + i_t}{1 + \pi_{t+1}} u'(C_{t+1}) \right]. \tag{9}$$

Households convey their stochastic discount factor $\Lambda_{t+\tau} = \beta^{t+\tau} u'(C_{t+\tau})$ to the firms they own, while regarding dividends and aggregate investments $X_t := \sum_{j=1}^{n} x_{jt}$ as given and exogenous for their decision.

2.4 Sticky real wage

We introduce different degrees of real wage stickiness to the model. Let $g \in [0, 1]$ denote an exogenous parameter that indicates the flexibility of real wages. The labor supply function can be expressed as

$$w_t = (w_t^f)^g (w_{ss})^{1-g} \tag{10}$$

where

$$w_t^f := v'(N_t)/u'(C_t)$$

denotes a frictionless real wage. This labor supply function nests the polar cases of perfectly flexible real wage $g = 1$ and constant real wage $g = 0$. w_{ss} represents the steady-state marginal rate of substitution, such that there is no steady-state distortion in the labor supply of households.

2.5 Monetary policy and market clearing

We assume that monetary policy sets a risk-free nominal rate at

$$1 + i_t = (1 + r_{ss})(1 + \pi_t)^{\phi} \tag{11}$$

with Taylor principle $\phi > 1$, and the steady-state real interest rate r_{ss} is given by $1/\beta - 1$.

A risk-free asset is supplied at net zero: $A_t = 0$. The market-clearing conditions for labor and final goods are:

$$N_t = L_t, \tag{12}$$

$$Y_t \left(1 - \frac{\psi_P}{2}\pi_t^2\right) = C_t + X_t. \tag{13}$$

2.6 Recursive equilibrium when $n \to \infty$

In the remainder of this section, we characterize the steady state and the equilibrium dynamics around the steady state in the limit case of the model as $n \to \infty$. A recursive equilibrium of the model in general form involves a dynamic mapping of firms' state distribution $F_t(a, s)$, which results in a curse of dimensionality for optimization behaviors. We assume the following for the firm's state variables to maintain tractability.

Assumption 1 *(i) Idiosyncratic productivity $a_{i,t}$ is i.i.d. across i and t and has finite support $\mathcal{A} = \{a(1), a(2), \ldots, a(H)\}$ with* $\max(a) - \min(a) < -(1-\delta)/\log\lambda$.

(ii) The initial value $s_{i,0}$ conditional on every value of $a_{i,0} \in \mathcal{A}$ is uniformly distributed over $[0, 1)$.

Under Assumption 1, $s_{i,t}$ is uniformly distributed in $[0, 1)$ for all t. This occurs because adding common shocks $(\log(\Phi_{t-1}K_t) - \log(\Phi_t K_{t+1}) - \log(1-\delta))$ and independent shocks $((\eta - 1)(\log a_{i,t} - \log a_{i,t-1}))$ to s_{it}, which is uniformly distributed over a circumference, keeps $s_{i,t+1}$ in the same distribution (see, e.g., Caballero and Engel, 1991). The support of a imposed by (i) simplifies the analysis, as firms do not choose to divest in a stationary equilibrium. Under this assumption, firms in a stationary equilibrium either choose a capital increase by λ or opt for inaction.

$F_t(a, s)$ remains at the stationary distribution F under Assumption 1. Thus, equation (8) implies that Φ_t is constant at $\Phi = \mathbb{E}^F[\tilde{a}\lambda^{\rho s}]^{-1/\rho}$, which leads to:

$$1 = \mathbb{E}_t \left[\frac{\beta u'(C_{t+1})}{u'(C_t)} \left(\frac{\kappa}{\Phi^{1-\rho}} \frac{\lambda^\rho - 1}{\lambda - 1} m_{t+1}^{1/\alpha} w_{t+1}^{1-1/\alpha} + 1 - \delta \right) \right].$$

Moreover, under Assumption 1, the law of motion for aggregate capital is expressed as

$$K_{t+1} = (1-\delta)K_t + A_X X_t, \tag{14}$$

where the constant parameter A_X adjusts the difference between aggregate investments X_t and the productivity-weighted average of capital K_t, depending solely on the productivity profile and exogenous parameters.[5]

In the limit of $n \to \infty$, the recursive equilibrium of $(Y_t, K_{t+1}, X_t, L_t, N_t, C_t, w_t, m_t, i_t, \pi_t)$ is determined by (1,2,3,8,9,10,11,12,13,14) under Assumption 1. We express $K_{t+1} = \Xi(K_t)$ for a mapping of aggregate capital that the recursive equilibrium establishes.

Equilibrium dynamics The recursive equilibrium system can be expressed as follows.

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \tag{GDP}$$

$$L_t = ((1 - 1/\eta)(1-\alpha)m_t/w_t)^{\frac{1}{\alpha}} K_t \tag{Labor Demand}$$

$$w_t = (v'(L_t)/u'(C_t))^g w_{ss}^{1-g} \tag{Labor supply}$$

$$C_t + X_t = \left(1 - \frac{\psi_P}{2}\pi_t^2\right) Y_t \tag{Goods market clearing}$$

$$K_{t+1} = (1-\delta)K_t + A_X X_t \tag{Capital accumulation}$$

$$1 = \mathbb{E}_t \left[\frac{\beta u'(C_{t+1})}{u'(C_t)} \frac{(1+r_{ss})(1+\pi_t)^{\phi}}{1+\pi_{t+1}}\right] \tag{Euler equation and Taylor rule}$$

$$1 = \mathbb{E}_t \left[\frac{\beta u'(C_{t+1})}{u'(C_t)} \left(\frac{\kappa}{\Phi^{1-\rho}} \frac{\lambda^{\rho} - 1}{\lambda - 1} m_{t+1}^{1/\alpha} w_{t+1}^{1-1/\alpha} + 1 - \delta\right)\right] \tag{Factor prices}$$

$$\pi_t(1+\pi_t) = \frac{\epsilon_c - 1}{\psi_P}(m_t - 1) + \mathbb{E}_t \left[\frac{\beta u'(C_{t+1})}{u'(C_t)} \frac{Y_{t+1}}{Y_t} \pi_{t+1}(1+\pi_{t+1})\right] \tag{Phillips curve}$$

The steady state of the above system of equations satisfies $\pi_t = 0$, $m_t = 1$, and $A_X X = \delta K$. Under flexible real wages ($g = 1$), the steady-state wage is determined by: $\left(\frac{1}{\beta} - 1 + \delta\right)^{\alpha} w^{1-\alpha} = \mathbb{E}^F \left[\lambda^{s\rho} a^{\eta-1}\right]^{1/(\eta-1)} \left(\frac{\lambda^{\rho}-1}{\lambda-1}\kappa\right)^{\alpha}$. Our model continuously approaches the divisible capital model when $\lambda \to 1$, as we see in Section 4.6.

[5] A_X is derived under Assumption 1 as follows. Let $s^*_{it} = (-\log(1-\delta) + \log k^*_{i,t+1} - \log k^*_{it})/\log\lambda$. Then, we have $\lambda^{s^*_{it}} = (1-\delta)^{-1} k^*_{i,t+1}/k^*_{it}$. By utilizing $k^*_{it} = a_{it}^{\eta-1}\Phi K_t$, we derive

$$\begin{aligned} X_t &= \iint_0^{s^*_{it}} (\lambda - 1)(1-\delta) k_{it} ds_{it} da_{it} = (\lambda-1)(1-\delta) \iint_0^{s^*_{it}} \lambda^{s_{it}} k^*_{it} ds_{it} da_{it} = (\lambda-1)(1-\delta) \int \frac{\lambda^{s^*_{it}} - 1}{\log\lambda} k^*_{it} da_{it} \\ &= \frac{(\lambda-1)(1-\delta)}{\log\lambda} \int ((1-\delta)^{-1} k^*_{i,t+1} - k^*_{it}) da_{it} = \frac{\lambda - 1}{\log\lambda} \Phi \mathbb{E}^F[a^{\eta-1}](K_{t+1} - (1-\delta)K_t), \end{aligned}$$

where the last equation holds since $a_{i,t+1}$ and $a_{i,t}$ follow F. Thus, we find that $A_X = \log\lambda/(\mathbb{E}^F[a^{\eta-1}]\Phi(\lambda - 1))$. Moreover, $\Phi = (\mathbb{E}^F[a^{\eta-1}](\lambda^{\rho} - 1)/(\rho\log\lambda))^{-1/\rho}$ by the uniform distribution of s_{it} under Assumption 1(ii).

Linearized dynamics around the steady state We examine a first-order perturbation of the equilibrium system around the steady state to analyze the dynamics. We denote $\tilde{X}_t$ as the proportional deviations of the variable X_t from its steady state value. The preference is specified as $u(C) = (C^{1-\sigma} - 1)/(1-\sigma)$ and $v(N) = \chi N^{1+1/\psi}/(1+1/\psi)$, where $\sigma > 0$ and $\psi > 0$. Consequently, we obtain the following system of equations.

$$
\begin{aligned}
\tilde{K}_{t+1} &= (1-\delta)\tilde{K}_t + \delta \tilde{X}_t \\
\alpha \tilde{K}_t + (1-\alpha)\tilde{L}_t &= \frac{C_{ss}}{Y_{ss}}\tilde{C}_t + \frac{X_{ss}}{Y_{ss}}\tilde{X}_t \\
\tilde{m}_t - \tilde{w}_t &= \alpha(\tilde{L}_t - \tilde{K}_t) \\
\tilde{w}_t &= g(\sigma \tilde{C}_t + (1/\psi)\tilde{L}_t) \\
\tilde{C}_{t+1} - \tilde{C}_t &= \frac{1-\beta(1-\delta)}{\sigma}\Big((1-\alpha)(\tilde{L}_{t+1} - \tilde{K}_{t+1}) + \tilde{m}_{t+1}\Big) \\
\pi_t &= \frac{\epsilon_c - 1}{\psi_P}\tilde{m}_t + \beta \pi_{t+1} \\
\sigma(\tilde{C}_{t+1} - \tilde{C}_t) &= \phi \pi_t - \pi_{t+1}
\end{aligned}
$$

3 Investment demand shocks in a finite economy

In the previous section, we characterized the recursive equilibrium with an infinite number of firms, $n \to \infty$. This section analyzes when n is finite. Due to the finite number of firms, each constrained by indivisible capital, aggregate investments generically deviate from those of an infinite number of firms. We will show in Section 4 that this deviation is quantitatively non-negligible even when n is large. The non-negligible deviation arises from an amplification effect of the number of firms engaging in investment spikes, which we call an *investment avalanche*. In Section 3, we begin by defining an investment demand shock. We then demonstrate impulse-response functions in a simplified version of the model, which indicate that the shock generates contemporaneous comovement of investment, consumption, and inflation.

3.1 Investment demand shocks: definition

In Section 2, we present a model with an infinite number of intermediate producers and derive a recursive equilibrium that pins down the aggregate capital path as $K_{t+1} = \Xi(K_t)$. The intermediate producers incur idiosyncratic productivity shocks but cannot generate aggregate stochastic fluctuations in this limiting case.

This section considers a large but finite n number of intermediate producers. Let K_t^n represent the aggregate capital in the finite n economy explicitly. Due to this finiteness, the idiosyncratic shocks to producers can deviate from the deterministic path established by Ξ and the initial capital K_t^n. This deviation generates aggregate stochastic fluctuations around the expected path.

We need two assumptions to facilitate the analysis of a finite economy model: a behavioral assumption to address the dimensionality issue arising from a finite number of firms and an equilibrium selection to manage a multiplicity of equilibria. To handle the latter, we select an equilibrium aggregate capital K_{t+1}^n that is close to the equilibrium aggregate capital predicted by the infinite model based on the current average capital, $\Xi(K_t^n)$, as discussed further in Section 4. Through this equilibrium selection, we choose the least volatile equilibrium path, precluding sunspot equilibria that arise from informational coordination.

The model economy features the profile of $(a_i, s_i)_{i=1}^n$ as a state, which introduces the curse of dimensionality with finite n firms as discussed by Krusell and Smith (1998). To address this issue, we make a behavioral assumption that agents use a stationary distribution F in the infinite economy instead of the actual finite profile of $(a_i, s_i)_{i=1}^n$ to form expectations. Consequently, households anticipate that the economy's future path will be determined by Ξ. Furthermore, we assume that intermediate firm j follows its threshold policy under F. Specifically, firm j follows a threshold rule (6) with $\Phi_t = \Phi$. Thus, intermediate firms respond to the realizations of K_{t+1}^n and $a_{i,t+1}$, but not to the entire profile $(a_{i,t+1}, s_{i,t+1})_{i=1}^n$. We will verify that the deviation between the expected and actual $(a_i, s_i)_{i=1}^n$ is small in Section 4.4.

Thus, the expected average investment is $X_t^e = (\Xi(K_t^n) - (1-\delta)(K_t^n))/A_X$ in an economy with n firms and predetermined aggregate capital K_t^n. An *investment demand shock* ϵ_t is then defined as the deviation of actual average investment from X_t^e:

$$\epsilon_t := X_t^n / X_t^e - 1,$$

where X_t^n represents the aggregate investment determined in the model with a finite number of n firms.

We establish the timing of the investment demand shock as follows. At the end of $t-1$, aggregate capital K_t^n has already been installed, and the recursive equilibrium determines the expected capital in $t+1$ as $K_{t+1}^e = \Xi(K_t^n)$. Therefore, the average investment for t expected at the end of $t-1$ is $X_t^e = (K_{t+1}^e - (1-\delta)K_t^n)/A_X$.

At the beginning of t, the actual aggregate investment X_t^n is determined in a model with n firms, which depends on the capital profile of these firms and the realization of idiosyncratic productivity shocks. The deviation of X_t^n from X_t^e constitutes an investment demand shock ϵ_t. After ϵ_t is realized, production and consumption in t occur.

We will demonstrate in Section 4 that ϵ_t, the investment demand shock caused by an avalanche, displays quantitatively significant variation. Thus, Section 4 offers a microfoundation for the aggregate investment demand shock ϵ_t.

3.2 Inflationary response to an investment demand shock: A simple case of constant real wages

We derive an analytical solution for the dynamics in a simple setup with constant real wages, specifically, $g = 0$. Assume that the economy was in its steady state at $t = 0$, and an investment shock $\epsilon_1 > 0$ occurs at $t = 1$. In period 2, the economy begins with a higher capital stock $\tilde{K}_2 > 0$. Because the economy possesses more wealth and higher labor productivity than in the steady state, the dynamics after $t = 2$ show a positive supply shock compared to the steady state, resulting in a consumption boom $\tilde{C}_t > 0$ and deflation $\pi_t < 0$ for $t = 2, 3, \ldots$.

We are interested in the response on impact at $t = 1$, prior to the capital increases. The linearized dynamics in Section 2.6 can be utilized to determine the response analytically.

$$\frac{K_{ss}^{\alpha} L_{ss}^{1-\alpha}}{C_{ss}}(1-\alpha)\tilde{L}_1 = \tilde{C}_1 + \frac{X_{ss}}{C_{ss}}\epsilon_1 \tag{15}$$

$$\tilde{m}_1 = \alpha \tilde{L}_1 \tag{16}$$

$$\pi_1 = \frac{\epsilon_c - 1}{\psi_P} \tilde{m}_1 + \beta \pi_2 \tag{17}$$

$$\tilde{C}_2 - \tilde{C}_1 = \frac{1}{\sigma} (\phi \pi_1 - \pi_2) \tag{18}$$

The investment demand shock propagates to inflation in this four-equation system (15–18) intuitively. The increase in investment demand ϵ_1 shifts out labor demand in (15) for fixed consumption. The rise in labor input decreases the marginal product of labor in the intermediate sector and raises the price of intermediate goods in (16), given the constant real wage. The increase in the intermediate cost $\tilde{m}_t$ results in inflation in the Phillips curve (17).

The procyclical effect on inflation may reverse if $\tilde{C}_1$ decreases significantly. However, this possibility can be ruled out if the future deflation π_2 is sufficiently small. We can obtain $\pi_1 > 0$ when $\pi_2 = 0$ and $\tilde{C}_2 > 0$ after some algebra involving the four equations.

Diminishing returns to labor and real wage rigidity establish a key environment for the propagation of an investment demand shock to inflation. If returns to labor inputs are constant, there is no channel (16) where an increase in labor inputs raises the marginal costs of final goods. Capital plays an essential role here, as evident from $\alpha > 0$ in (16). The sticky real wage is also important. In a general case where $g > 0$, the labor demand function is $\tilde{m}_t - \tilde{w}_t = \alpha(\tilde{L}_t - \tilde{K}_t)$ as described in Section 2.6. Thus, a decrease in real wages may facilitate an increase in labor input if wages are sufficiently flexible, alleviating the cost pressure on inflation.

3.3 Impulse-response functions

We numerically solve for impulse-response functions in a general case of $g > 0$, where real wages are sticky but not constant. We will compare the investment demand shock to an investment-specific technological (IST) shock in terms of the inflation responses. To achieve this, we allow A_X in the capital accumulation equation (14) to vary. This modification is accomplished by introducing an i.i.d. random variable a_t^X, which denotes the logarithmic difference from A_X.

We examine two versions of the IST shock. In the first version, we follow Justiniano, Primiceri, and Tambalotti (2010) to define a marginal efficiency of investment (MEI) shock as a transformation ratio of newly installed capital to purchased investment goods. The MEI shock is revealed to agents when the investment materializes as capital, i.e., in the next period after the investment purchase. An alternative version of the shock introduces a news shock in which MEI shocks that expand investment in $t = 2$ are revealed to agents in $t = 1$. The first version results in a straightforward lack of response in $t = 1$, while the second version leads to actions in $t = 1$ due to the arrival of the news.

Parameter values are calibrated as follows: we calibrate the model to annual data and set $\beta = 0.98$ and $\delta = 0.1$. The elasticity of substitution for the final wholesale sector is set at $\epsilon_c = 6$, while for the intermediate sector, it is set at a competitive level, $\eta = 30$. We set χ so that the steady-state labor supply equals 1. The lumpiness parameter is $\lambda = 1.2$, corresponding to the benchmark value of 20% used in the literature on lumpy investments (Cooper, Haltiwanger, and Power, 1999; Gourio and Kashyap, 2007). The degree of wage flexibility is set at $g = 0.12$ at the annual frequency. At this value, the volatility of real wages generated by the model is compatible with the U.S. experience. The real weekly earnings of wage and salary workers in NIPA for 1979Q1-2024Q3 exhibit 0.44% standard deviations for the quarterly growth rate, whereas our model calibrated at the quarterly frequency (with the corresponding quarterly rate $g = 0.03$) generates 0.73% standard deviations for $\log w_t$. Table 1 summarizes the parameter values.

α	β	δ	ψ	ϵ_c	η	σ	ψ_P	λ	ϕ	g
0.36	0.98	0.1	0.5	6	30	3	30	1.2	1.2	0.12

Table 1: Calibrated values of parameters at the annual frequency

To compute the impulse-response of an investment demand shock, we assume that investment demand X increases by 4.9%, representing the standard deviation of the annual growth rate of real gross fixed capital formation in the U.S. from 1972 to 2024. This shock to X results in capital in the next period being 0.49% higher than the steady-state level. When computing the impulse-response of an IST shock without news, we also set A_X to increase by 4.9%, leading to capital in the next period being 0.49% above the steady state. In the case of an IST shock with news, the value of X_t adjusts in response to the news. Therefore, in this experiment, we analyze the IST shock on A_X to achieve a 0.49% increase in capital.

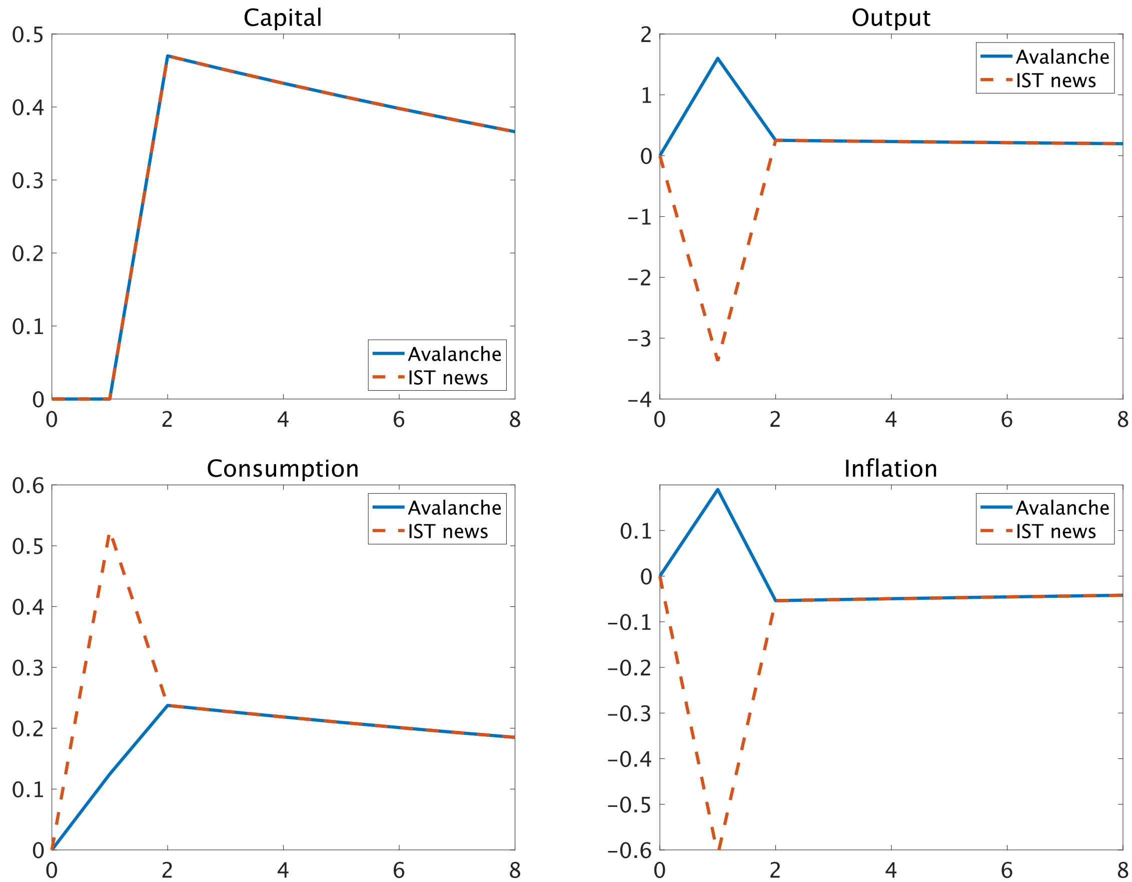

Figure 2: Impulse response functions (annual) for capital, output, consumption, and inflation rates to a one-standard-deviation shock (4.9%) in investment. The horizontal axis represents years, while the vertical axis shows a percentage deviation from the steady state, except for inflation, which presents a percentage point deviation.

Figure 2 shows the impulse responses for an investment demand shock depicted with solid lines, while those for an IST news shock are represented with dashed lines. The impulse responses for an IST shock without news are identical to those for an IST news shock from $t = 2$ onward, and no responses occur in $t = 1$.

The top left panel shows that the capital transition is identical across the two experiments: capital increases by 0.1% in the first period following the shock and gradually converges back to the steady state. The bottom left figure indicates that the consumption path is the same after $t = 2$ between the two experiments. However, consumption differs at $t = 1$. In line with our analysis of $g = 0$, an investment demand shock triggers a positive reaction in contemporaneous consumption. The top right panel displays output responses. For an avalanche shock, output increases upon shock because both investment and consumption rise, while an IST news shock decreases output due to a decline in the purchase of investment goods following the shock. The output response is facilitated by labor input, which fluctuates in reaction to labor demand under real wage rigidity.

The bottom-right plot illustrates inflation's response. The inflation rate falls below the steady-state level in all experiments after $t = 2$. This indicates the convergence toward the steady state following a positive supply shock from the capital increase. After the capital rises, the gradual decline in capital is associated with a slow decrease in consumption. Hence, during the transition, the real interest rate must increase toward the steady-state level, leading to disinflation upon the impact of the capital increase and returning to steady-state inflation afterward.

A stark contrast emerges in the inflation response between an investment demand shock and an IST news shock. Consistent with our previous analysis for $g = 0$, the inflation rate reacts positively to an investment demand shock in the period when the shock occurs. This procyclical effect of the investment demand shock on inflation can be understood by examining the equilibrium conditions. The investment demand shock negatively affects consumption in the goods market clearing condition and positively in the labor demand condition. The first direct effect on consumption is tempered by the wealth effect, where households, anticipating an increase in capital in the future, seek to boost their consumption level immediately. The second direct effect decreases the marginal product of labor and raises the real price of intermediate goods when the real wage is rigid. An increase in marginal unit labor costs is passed through to intermediate prices, which in turn drives up the inflation rate.

We observe that the positive consumption response and the negative inflation response occur due to the IST news shock, which is straightforward to interpret. The wealth effect from the positive shock in the investment sector leads to an increase in consumption once the shock is revealed. The immediate response in consumption is stronger than that of an investment demand shock because the anticipated increase in capital does not require any investment resources. This significant immediate rise in consumption results in negative consumption growth in the following period, which depresses the real interest rate and thereby lowers the inflation rate according to the Taylor principle of the monetary policy response function.

The analysis in this section demonstrates that an investment demand shock leads to a positive contemporaneous response in inflation. An investment demand shock increases future capital, resulting in a wealth effect that boosts consumption. Furthermore, the rise in investment and consumption shifts labor demand outward. With sticky real wages, the increased labor demand raises unit labor costs and the costs of intermediate goods for final goods producers. This marginal cost effect causes contemporaneous inflation.

3.4 Time to build

Previous results on impulse-response functions in Figure 2 demonstrated a short-lived response for inflation. A richer response is achieved in a more realistic framework where capital increases only gradually following a decision to invest in a spike. To this end, we expand the model to incorporate time-to-build. The extended model is detailed in Appendix C.2. Here, we present only the impulse-response function results from this extended model.

Figure 3 displays the impulse-response functions when an investment demand shock occurs at $t = 1$. Time is measured quarterly in this setup. The parameter values that depend on the time unit are adjusted for the quarterly framework: $\eta = 0.995$, $\delta = 0.025$, $g = 0.03$, and $\psi_P = 80$, while other parameters remain unchanged from Table 1. It is assumed that capital grows linearly for six quarters until reaching completion at $t = 7$.

The top left panel of Figure 3 shows a gradual increase in capital over six quarters. The bottom left panel indicates a steady rise in consumption, reflecting the growth in production capacity of this economy during the build-up. The top right panel depicts output, revealing a significant increase during the transition periods when real wage rigidity is strongly binding and labor input rises considerably.

The bottom right panel illustrates an inflation path. Inflation spikes on impact and gradually decreases to a slightly deflationary level at $t = 7$, after which it stabilizes. The

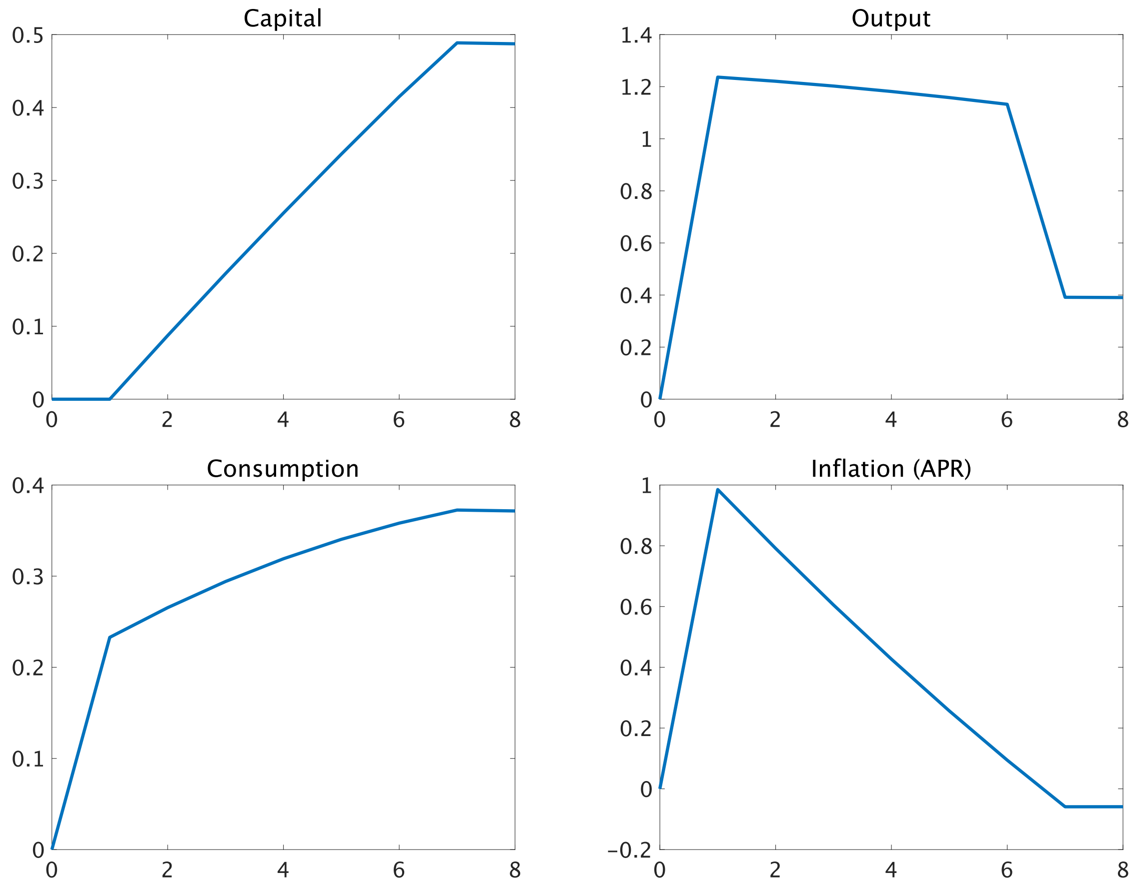

Figure 3: Impulse response functions (quarterly) for capital, output, consumption, and inflation rates with six quarters of time-to-build. The horizontal axis represents quarters. The impulse is one standard deviation of the growth rate of real investment in a quarterly frequency, which is 3.7% for 1947Q2–2024Q4 in the U.S.

disinflation during the build-up period is a result of increasing consumption during that time. Thus, an investment demand shock positively influences both consumption and inflation. In this model, the inflation effect manifests earlier than the consumption effect. Therefore, an investment demand shock positively impacts both consumption and inflation, with the inflation effect occurring before the consumption effect.

Although a fully quantitative analysis is beyond the scope of this paper, the propagation mechanism highlighted in this section demonstrates quantitatively promising results. A back-of-the-envelope calculation using (15-18) may be useful. When a one-standard-deviation investment shock occurs, capital in the next period increases by approximately 0.5%, raising permanent income and future consumption by at most 0.5%. Due to consumption smoothing (18), this results in an increase in contemporaneous consumption of 0.5% or less, depending on the monetary policy parameter. The rise in consumption and the investment shock leads to an increase in final goods demand of $5 \times (I/Y) + 0.5 \times (C/Y) = 1.4\%$ at most. The output increase is supported by a rise in labor inputs of $1.4/(1-\alpha) = 2.1\%$ at most. We note that the labor response exceeds empirical regularities, while the consumption response is smaller. However, these discrepancies can be attributed to factors not accounted for in our model, such as capital utilization and hand-to-mouth households.

3.5 Policy analysis

The investment demand shock contrasts with an IST news shock regarding inflation responses upon impact. Additionally, the demand shock has policy implications that differ from those of the IST shocks. The investment demand shock results in inefficiently high hours worked due to real wage rigidity. This leads to an increase in inflation, prompting households to work and consume more, which creates distortions in the labor supply that need to be minimized. This increase in inflation is significantly smaller in the IST case.

We illustrate this point through an exercise that considers an optimal inflation rate determined by discretionary monetary policy, without concerns about future commitment issues. In period t, the monetary authority faces an investment demand shock, or an IST shock. Capital in $t+1$ will be above the steady-state level, and the recursive equilibrium determines all $t+1$ variables known to the monetary authority. Let's consider a choice of π_t that maximizes the utility of representative households:

$$\max_{C_t, L_t} u(C_t) - v(L_t) + \beta V\left(K_{t+1}, Y_{t+1}, \pi_{t+1}\right)$$

subject to all equilibrium conditions, including

$$K_{ss}^{\alpha} L_t^{1-\alpha} = C_t + \frac{\psi_P}{2} \pi_t^2 Y_{ss} + X_{ss}\left(1 + \epsilon_t\right)$$

in the constant real wage case, $g = 0$.

Since all $t+1$ variables are given, only the allocation at t is of concern. Then,

$$\pi_t(1+\pi_t) = \frac{\varepsilon_P - 1}{\psi_P}\Big(\frac{w_{ss}}{(1-\alpha)\,K_{ss}^{\alpha}}L_t^{\alpha} - 1\Big) + \beta\mathbb{E}_t\Big(\pi_{t+1}(1+\pi_{t+1})\frac{Y_{t+1}}{Y_t}\Big). \tag{19}$$

This equation implicitly defines an equilibrium relationship Ψ such that:

$$L_t = \Psi(\pi_t).$$

Using Ψ, the maximization problem for the monetary policymaker is:

$$\max_{\pi_t} u\left((K_{ss}^{\alpha}\Psi(\pi_t)^{1-\alpha} - \frac{\psi_P}{2}\pi_t^2 Y_{ss} - X_{ss}(1+\epsilon_t))/n\right) - v(\Psi(\pi_t)/n) + \beta V(K_{t+1}, Y_{t+1}, \pi_{t+1}).$$

With writing $\mathrm{MPL}_t := (1-\alpha)\,K_{ss}^{\alpha}L_t^{-\alpha}$, the first-order condition for optimal π_t is:

$$u'(C_t) \times \mathrm{MPL}_t - v'(L_t) = \frac{u'(C_t)\psi_P \pi_t Y_{ss}}{\Psi'(\pi_t)}.$$

If $\pi_t = \pi_{t+1}$ is chosen, (19) yields (up to the first order) $L_t = L_{ss}$ and $\mathrm{MPL}_t = \mathrm{MPL}_{ss}$, but $C_t < C_{ss}$. As a result, we obtain:

$$\begin{aligned}
&u'\left(C_t\left(\pi_t = \pi_{t+1}\right)\right) \times \mathrm{MPL}_t\left(\pi_t = \pi_{t+1}\right) - v'\left(L_t\left(\pi_t = \pi_{t+1}\right)\right) \\
&= u'\left(C_t\left(\pi_t = \pi_{t+1}\right)\right) \times \mathrm{MPL}_{ss} - v'(L_{ss}) > 0.
\end{aligned}$$

Thus, the policymaker aims for households to consume and work more. To achieve this allocation, $\pi_t > \pi_{t+1}$ must be chosen. Since the deflationary effect of capital K_{t+1} on π_{t+1} is quantitatively small, the optimal inflation rate π_t remains positive when the labor wedge $\mathrm{MPL}_{ss}u'(C_t)/v'(L_{ss})$ is sufficiently large.

Indeed, if the inflation rate is 0 in the first period, labor and output are very close to their steady-state values due to the New-Keynesian Phillips curve (19). As investment increases and output remains fixed, consumption declines. Consequently, households' first-order condition regarding their labor supply is not fulfilled: $u'(C_t)\mathrm{MPL}_t > v'(L_t)$. In other words, agents want to work more to consume more. Positive inflation stimulates the economy, leading to an increase in labor demand since the real wage is fixed. Thus, the monetary authority has an incentive to allow positive inflation to counteract the negative effect of an investment demand shock on consumption.

In the case of an IST shock, the investment amount required to generate the same capital stock as in an investment demand shock is smaller because agents correctly expect an increase in investment productivity. Therefore, the distortionary effects caused by real wage rigidity are significantly smaller than those present during an investment demand shock.

4 Microfoundation of investment demand shocks

This section analyzes an investment demand shock, ϵ_t. The investment demand shock is the gap between the equilibrium average investment, X_t^n, in an economy with n and infinite firms. It is expressed as $\epsilon_t := X_t^n/X_t^e - 1$. This section demonstrates analytically and numerically that ϵ_t exhibits significant fluctuations.

It is commonly believed that idiosyncratic productivity shocks would result in minimal aggregate fluctuation in ϵ_t due to the diversification effect. According to the central limit theorem, the standard deviation of an average of independent n shocks decreases at a rate of $1/\sqrt{n}$. For instance, the idiosyncratic shocks of one million firms, with a 10% standard deviation, produce aggregate volatility of $10\%/\sqrt{10^6} = 0.01\%$ standard deviation, which does not match the order of magnitude of business cycle fluctuations.

The diversification argument for ϵ_t does not hold in our model with n firms. First, a firm's investment is a non-linear function of aggregate capital due to indivisibility. Second, firms' investment decisions are complementary in equilibrium. A firm's choice to invest increases the aggregate capital, which raises the demand for all firms and pushes up the threshold for their investment decisions. Thus, a firm's lumpy investment increases the likelihood of other firms' investments and triggers an avalanche of investment spikes within a period. The aggregate investment demand shock ϵ_t is determined by the size of the investment avalanche.

4.1 Complementarity of lumpy investment decisions

The complementarity exhibited by firms' investment decisions in equilibrium plays a central role in our analysis. We now formally define the degree of complementarity. To understand what determines the degree of complementarity, we generalize a model to decreasing returns to scale. An intermediate firm's production function is extended to $y_{jt}^m = a_{jt}(k_{jt}^{\alpha} l_{jt}^{1-\alpha})^{\theta}$, where $\theta = 1$ corresponds to the model in Section 2. Analyses in this section naturally generalize to the decreasing returns to scale with $\theta < 1$ as we show in Appendix A. The optimal threshold in the general case is $k^*_{j,t+1} = \tilde{a}_{j,t+1}\Phi_t K^{\tilde{\theta}}_{t+1}$, where $\tilde{a}_{jt} := a_{jt}^{\frac{1-1/\eta}{1-\theta+\theta/\eta}}$ and $\tilde{\theta} := \frac{(\alpha\theta)/\eta/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}$.

Consider the steady-state equilibrium of the model with an infinite number of firms. Let Δ denote the forward difference of a variable. In partial equilibrium, employing the threshold policy, the state variable $s_{it} = (\log k_{it} - \log k^*_{it})/\log\lambda$ evolves as Using the threshold policy in partial equilibrium, the state variable $s_{it} = (\log k_{it} - \log k^*_{it})/\log\lambda$ evolves as

$$s_{i,t+1} = s_{it} + \frac{\log(1-\delta) - \Delta\log\tilde{a}_{it} - \tilde{\theta}\Delta\log K_t}{\log\lambda} \tag{20}$$

if $s_{i,t+1} > 0$. At the beginning of a period, depreciation $1-\delta$ and productivity shocks $\Delta\log\tilde{a}_{it}$ are realized. Since we start from the steady state of an infinite model, $\Delta\log K_t = 0$ holds. Therefore, firms with $s_{it} \le -\frac{\log(1-\delta)-\Delta\log\tilde{a}_{it}}{\log\lambda} =: s^*_{it}$ invest and transition to $s_{i,t+1} = 1+s_{it}-s^*_{it}$, while other firms' $s_{i,t+1}$ is positive and determined by (20). s^*_{it} denotes the threshold for investment.

Since $a_{i,t}$ has a finite support $\mathcal{A} = \{a(1),\ldots,a(H)\}$ as stated in Assumption 1, firms are classified into H^2 groups based on the realization of $(a_{i,t}, a_{i,t+1})$. Specifically, firms that experience $a_{i,t} = a(h_0)$ and $a_{i,t+1} = a(h_1)$ belong to group $h = h_0 + (h_1-1)H$. Let $\Delta\log\tilde{a}(h) = \log\tilde{a}(h_1) - \log\tilde{a}(h_0)$, $s^*(h) = -(\log(1-\delta) - \Delta\log\tilde{a}(h))/\log\lambda$, and let $\omega(h)$ represent the measure of firms in group h relative to the total number of firms.

Now, we examine a perturbation in which a $\nu(h)$ measure of group-h firms located in $[s^*(h), s^*(h)+\nu(h))$ also invests. Since s is uniformly distributed, the firms in this interval have a measure of $\omega(h)\nu(h)$. The increase in the firms' investment leads to $\Delta K_{t+1} > 0$, reducing s_{jt} of other firms by ν'. Since s_{jt} is uniformly distributed, the measure of firms induced to invest due to the reduction in s_{jt} is ν'. Thus, we define the degree of complementarity as $\vartheta(h) := \lim_{\nu(h)\to 0}\nu'/(\omega(h)\nu(h))$. The complementarity $\vartheta(h)$ indicates the number of firms induced to invest by an investment from a firm in group h. Consequently, we derive the following property.

Proposition 1 *The degree of complementarity when a firm in group h invests is*

$$\vartheta(h) = \tilde{\theta}\frac{\tilde{a}(h_1)}{\mathbb{E}^F[\tilde{a}]},$$

where $\tilde{\theta} = \frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}$. The expected value $\vartheta := \sum_h \omega(h)\vartheta(h)$ equals $\tilde{\theta}$, increasing in θ for $\theta \le 1$, and converges to 1 as $\theta \to 1$.

The proof is provided in Appendix B.1.

The fraction of firms induced to invest by an investment from a randomly selected firm across groups is denoted by ϑ. Since ϑ is the expected value of $\vartheta(h)$ across h, an immediate consequence of Proposition 1 is that $\vartheta = 1$ under constant-returns-to-scale technology $\theta = 1$. This indicates that when an additional firm located at s^*_{it} invests, one more firm, on average, is induced to invest due to the positive dependence of the threshold k^* on aggregate capital K.

4.2 Investment avalanche

Due to the nonlinearity arising from indivisible capital, the model can exhibit multiple equilibria. We select an equilibrium that is close to the one determined by the model with an infinite number of firms. To find the equilibrium K^n_{t+1} that is close to $\Xi(K_t)$, we utilize the best response dynamics of firms. We define a firm's best response as follows.

$$k'_i = \gamma(a_i, k_i, K^e) = \begin{cases} k_i/\lambda & \text{if } k_i \geq \lambda \tilde{a}_i \Phi(K^e)^{\tilde{\theta}} \\ \lambda k_i & \text{if } k_i < \tilde{a}_i \Phi(K^e)^{\tilde{\theta}} \\ k_i & \text{otherwise} \end{cases}$$

An aggregate response function is defined as a mapping from an expected average capital to a new aggregate capital:

$$K' = \Gamma(K^e; (a_i, k_i)_i) = \left(\sum_{i=1}^{n} \left(a_i^{1/(\alpha\theta)} \gamma(a_i, k_i, K^e) \right)^{\rho} /n \right)^{1/\rho}.$$

Next, we define an equilibrium finding process that we call an investment avalanche.

1. Capital is depreciated as $k^d_i = (1-\delta)k_{i,t}$. Productivity profile $(a_i)_i$ realizes. The expected average capital is set at $K^e = \Xi(K_t)$.

2. Update capital profile and aggregate capital by $k^0_i = \gamma(a_i, k^d_i, K^e)$ and $K_0 = \Gamma(K^e; (a_i, k^d_i)_i)$. Stop if $K_0 = K^e$. Otherwise, set $u = 0$ and continue.

3. Update capital profile and aggregate capital $k^{u+1}_i = \gamma(a_i, k^u_i, K_u)$ and $K_{u+1} = \Gamma(K_u; (a_i, k^u_i)_i)$. Stop if $K_{u+1} = K_u$. Otherwise, set u to $u+1$ and repeat this step unless $k^u_i \neq k^0_i$ for all i.

Figure 4 illustrates the avalanche process. An investment avalanche starts with the realization of idiosyncratic productivity shocks and capital depreciation. The initial expectation of future capital K^e_{t+1} sets the extensive margin of firms that adjust capital in response to productivity shocks and depreciation. Equilibrium is reached when the resulting aggregate capital equals K^e_{t+1}. If not, successive capital adjustments occur until equilibrium is achieved.

The selected equilibrium is close to K^e in that there is no equilibrium aggregate capital with $k_i \in [k_i^*, \lambda k_i^*)$ between the selected K^n and K^e_{t+1}. This property holds due to the characteristics of the best response dynamics (Vives, 1990). We will demonstrate in the next section that the case of all firms adjusting (i.e., $k_i \neq k_i^o$ for all i) has a probability of zero as $n \to \infty$.

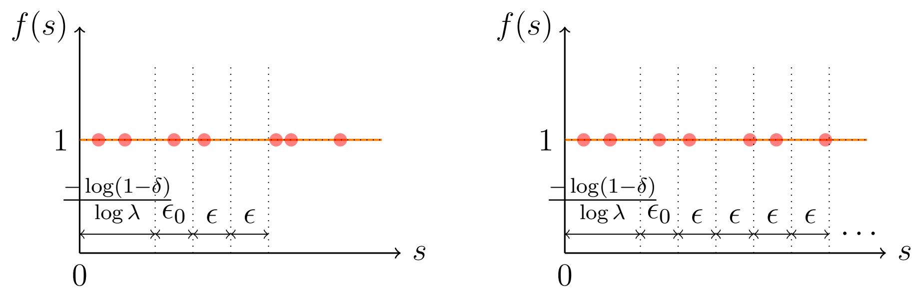

Figure 4: Investment avalanche. The evolution of profile (s_{it}) is driven by productivity shocks and capital depreciation. At the beginning of a period, depreciation (with productivity shocks omitted in the graph) induces investments. If the realized K exceeds K^e, a positive avalanche ensues. In the left panel, the avalanche stops in the second round. In the right panel, a slight change in profile (s_i) leads to a significant change in the resulting avalanche size.

4.3 Non-negligible fluctuations of investment avalanches

We demonstrate non-negligible fluctuations of the investment avalanche both analytically and numerically. Our analysis extends Nirei (2015) to an economy with sticky pricing and time-to-build, and enhances the analytical characterization using results from Nirei and Scheinkman (2024).

We use ϑ defined in the infinite model to characterize the asymptotic distribution of the number of firms involved in an investment avalanche. We consider an experiment in a model with a finite number n of firms. As in Proposition 1, we impose Assumption 1 that s_i follows a uniform distribution over $[0, 1)$ and that a_i and s_i are orthogonal at F. First, we draw a profile $(a_i, s_i)_{i=1}^n$ from F^n. This profile determines a capital profile $(k_i)_i$ by $k_i = \lambda^{s_i} k_i^*$ and $k_i^* = \tilde{a}_i \Phi K^{\tilde{\theta}}$, where K is the steady-state average capital in an infinite model. Firms expect K^e, the average capital for the next period, to be the same as K.

Then, we let capital depreciate at δ, and determine the next-period capital profile using the avalanche algorithm. In step 2, firms with state $s_i \leq s_i^*$ reach the threshold and choose to invest. Let $\epsilon_0^n = \log(K_0) - \log K^e$ represent the gap between the aggregate reaction and the expected capital.

Now, suppose that $\epsilon_0^n > 0$ occurs. In other words, the gross investments of firms that fall below the threshold due to the direct effect of depreciation exceed expectations. This raises the threshold, causing firms with $s_i^* < s_i \leq s_i^* + \tilde{\theta}\epsilon_0 / \log\lambda =: s_{i0}^*$ to fall below the threshold in step 3. Let Z_0^n represent the set of these firms, and let $z_0^n := |Z_0^n|$ denote their number. If $z_0^n > 0$, this increases aggregate capital by an increment denoted as ϵ_1^n. Consequently, a new set of firms with $s_{i0}^* < s_i \leq s_{i0}^* + \tilde{\theta}\epsilon_1^n / \log\lambda =: s_{i1}^*$ are induced to invest in the first iteration of step 3. Let Z_1^n represent the set of these firms, and let $z_1^n := |Z_1^n|$.

We repeat this process by defining ϵ_u^n, s_u^n, Z_u^n, and z_u^n for $u = 0, 1, \ldots, U$, where U denotes the first $u > 0$ such that $z_u^n = 0$. The size of an investment avalanche is defined as the number of firms involved in the process: $L^n := \sum_{u=0}^{U} z_u^n$. When $\epsilon_0^n < 0$, we can similarly define a process for retracting the investment decisions of the firms that initially invest due to the direct effect of depreciation. Since $(a_i, s_i)_i$ is stochastic, L^n is also stochastic. We are interested in the distribution of L^n, which characterizes the distribution of $\Delta \log K$ in this experiment.

Since s_i are i.i.d. random variables and the sets of newly investing firms Z_u^n are mutually exclusive, z_u^n represents a stochastic process. The probability function for z_{u+1}^n, conditional on z_u^n, follows a binomial distribution with a probability of $\tilde{\theta}\epsilon_u^n / \log\lambda$ and a population of $n - \sum_{v=0}^{u} z_v^n$. Therefore, $(z_u^n)_{u>0}$ constitutes a branching process in which each of the "parents" z_u^n produces "children" with a mean of $(n - \sum_{v=0}^{u} z_v^n)\tilde{\theta}\epsilon_u^n / (z_u^n \log\lambda)$. L^n represents the cumulative sum of this branching process with initial parents z_0^n until no children are born.

We demonstrate in Appendix B.2 that the mean of z_{u+1}^n conditional on $(z_v^n)_{v=0}^u$, which follows a binomial distribution with mean $(n - \sum_{v=0}^u z_v)\tilde{\theta}\epsilon_u^n / \log \lambda$, converges to ϑz_u^n as $n \to \infty$, where ϑ is the degree of complementarity defined in Section 4.1. Thus, as $n \to \infty$, the binomial distribution converges to a Poisson distribution with mean ϑz_u^n, which is equivalent to a z_u^n-convolution of a Poisson distribution with mean ϑ. Let L denote the cumulative sum of a Poisson branching process $(z_u)_u$, in which each parent bears a random number of children that follows a Poisson distribution with mean ϑ. Then, we derive the following properties using the results of Nirei and Scheinkman (2024): (i) L^n converges to L in total variation (Proposition 6), (ii) L conditional on z_0 follows the Borel-Tanner distribution with parameter ϑ (Proposition 8a), (iii) Since z_0 also follows a Poisson distribution, the unconditional distribution of L is represented as a Generalized Poisson distribution (Proposition 8b), (iv) Complementarity ϑ determines the shape of the tail of distributions. In particular, when $\vartheta \to 1$, the dispersion index of L diverges to infinity (Proposition 8d).

The distribution of ϵ_0^n adheres to the diversification effect. The number of firms in group h that invest in step 2—affected by the depreciation δ and productivity shocks $\Delta \log a(h)$—follows a binomial distribution with probability $s^*(h) = (-\log(1-\delta) + \Delta \log \tilde{a}(h)) / \log \lambda$ and population $\omega(h)n$. According to the central limit theorem, the number of initially investing firms, scaled by $1/\sqrt{n}$, asymptotically follows a normal distribution with finite variance determined by $(s^*(h)(1 - s^*(h)))_h$. Given that $|z_0^n|$ follows a binomial distribution with $|\epsilon_0^n|$ and population scaling as n, the mean of $|z_0^n|$ scales as $\sqrt{n}$.

Note that the sum L of a branching process, given an initial value z_0, is the z_0-times convolution of L conditional on the initial value 1. Thus, L^n asymptotically approaches the z_0^n-times convolution of L conditional on 1. The effect on K^n/n is determined by L^n/n. The initial value z_0/n scales as $1/\sqrt{n}$, in line with the diversification effect, while each instance of z_0 generates L conditional on 1, showcasing large volatility. Therefore, the significant fluctuation of K^n/n results from the fat-tailed distribution of L.

Finally, we note that the avalanche process examined here identifies the closest fixed point in the direction of the initial gap ϵ_0^n: $\text{sign}(K^n - K^e) = \text{sign}(\epsilon_0^n)$. It is possible that a fixed point in the opposite direction is closer to K^e. As argued in Nirei (2015), the distance from K^e to this fixed point can be characterized using the investment avalanche process.

Therefore, the smallest equilibrium fluctuation $|\log K^n - \log K^e|$ is the minimum of two avalanches. We investigate the distribution of the smallest fluctuation through numerical simulations in the next section.

4.4 Numerical analysis of investment avalanches

We examine the properties of investment avalanches through numerical simulations, focusing on a flexible real wage case where $g = 1$. First, we determine the steady state of an economy with an infinite number of firms. Next, we simulate the performance of an economy with a finite number of firms, denoted as n. The time unit is a quarter. The recursive equilibrium mapping Ξ determines the expected aggregate capital for each period. By using the computed results for $K^e_{t+1} = \Xi(K^n_t)$ and applying a first-order approximation of the dynamics, we set that $\log K^e_{t+1}$ follows a first-order autoregressive process with a persistence of 0.975, indicating that the half-life period of a deviation from the steady state is 20 quarters. We assume that firm-level productivity follows a logarithmic AR(1) process with a persistence of $\rho_a = 0.9$ and an i.i.d. shock with a standard deviation of $\sigma_a = 0.03$. The number of firms is set at $n = 30000$, reflecting the number of firms included in the Japanese business survey (BSJBSA).

We incorporate the time-to-build into the model; specifically, an investment gestation lag exceeds one period. Time-to-build allows the model to generate an autocorrelated series of aggregate investment. This happens for a straightforward reason. Let $J \geq 1$ represent the time to build. The investment is determined J periods in advance, and its purchase is spread over the $J - 1$ periods. In each period, firms' investment decisions lead to an investment avalanche related to the J-period ahead capital. Consequently, aggregate investment becomes the weighted average of past investment avalanches, resulting in autocorrelation. We set the time-to-build to $J = 6$ quarters. Other parameter values remain unchanged from those in Section 3.4.

Figure 5 displays the simulated time series of aggregate investments. The standard deviation of the fluctuations is 3.5%, and the autocorrelation coefficient stands at 0.74. It is important to note that the exogenous shocks applied to the model are firm-level independent productivity shocks with $\sigma_a = 0.03$. Thus, the simple average of productivity shocks results in $\sigma_a/\sqrt{n} \approx 0.017\%$ standard deviation, which cannot account for the 3.5% standard deviation of aggregate investment. The aggregate fluctuations stem from the evolution of the capital profile, which determines the magnitude of investment avalanches.

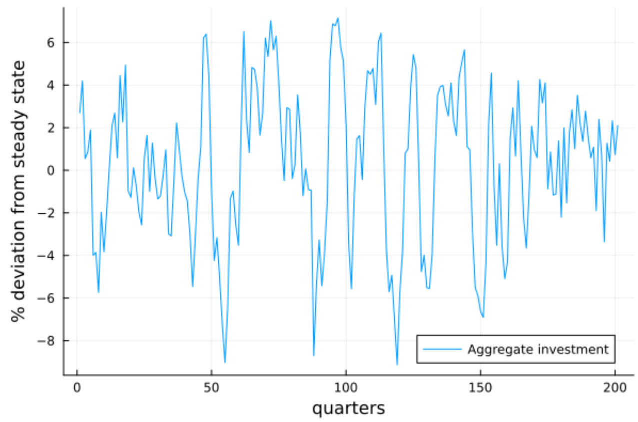

Figure 5: Simulated time series of aggregate investments

The size of investment avalanches is determined by the number of firms involved in the avalanche following the initial response to capital depreciation and productivity shocks. Figure 6 displays the distribution of investment avalanches in a simulated time series for 1000 quarters. We observe significant fluctuations in the ratio of investing firms to n. The distribution is well approximated by a normal distribution, indicating that the initial number of firms responding to capital depreciation and productivity shocks adheres to the central limit theorem.

The number of firms in an investment avalanche and the lumpiness λ determine the magnitude of aggregate investment made by firms during a given period. In this simulation, the standard deviation of the aggregate investment amount determined in a period is 10.8%. This amount is evenly distributed over the time-to-build periods J. The aggregate investment in a period is the sum of the overlapping investments made in the current and the previous $J - 1$ periods. Therefore, through diversification, the volatility of aggregate investments is lower than that of the aggregate investment amounts determined in a period.

Finally, we examine the distributions of (s_{it}, a_{it}) over time periods. In our analytical characterizations, we assumed that the distribution of s remains consistent with a uniform distribution. The simulation results support this assumption. We calculated the first four

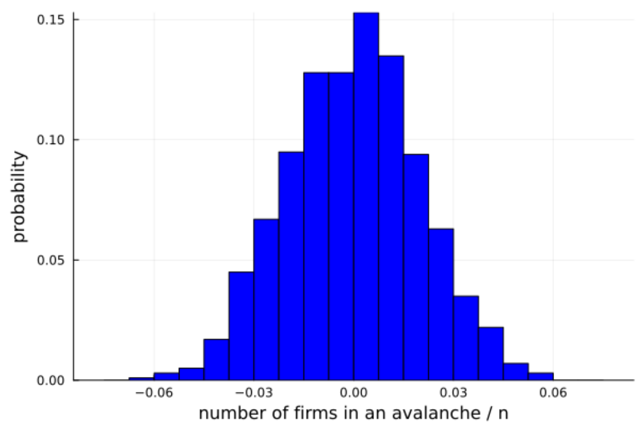

Figure 6: Histogram of the size of investment avalanches in the simulations

non-centered moments of $(s_{it})_i$ for each t. During a simulation lasting 1000 quarters, the mean deviations from the population moments do not exceed 0.02%. Furthermore, the maximum deviation of the first moment from that of the uniform distribution is 1.1%, and it remains below 2.5% for all four moments. The average correlation coefficient between profiles (s_i) and (a_i) is 0.14% across 1000 sample periods.

4.5 Welfare loss associated with investment avalanches

We stipulate that the intermediate producer employs a threshold rule (6) with stationary expectation Φ on the sufficient statistics of expected factor prices. A rationale for this behavioral assumption is that the sufficient statistic, Φ_t, equals Φ in equilibrium in any state of an economy with an infinite number of firms and constant returns to scale under Assumption 1. However, in an economy with n firms, an intermediate producer may profit by deviating from this behavioral rule.

Φ_t represents an expectation for a function of m_{t+1}, w_{t+1}, and Λ_{t+1}/Λ_t. The current investment shock, ϵ_t, impacts K_{t+1}, along with the expectations for m_{t+1}, w_{t+1}, and the consumption path that determines Λ_{t+1}/Λ_t, whereas investment shock in the next period, ϵ_{t+1}, influences the realization of $m_{t+1}, w_{t+1}, \Lambda_{t+1}$. We will examine these effects individually.

First, factor prices (m_{t+1}, w_{t+1}) influence aggregate demand (2) and the operating surplus of the intermediate producer. Therefore, if the intermediate producer knows the investment avalanche ϵ_{t+1} at time t and can forecast m_{t+1} and w_{t+1}, along with Y_{t+1}, it may enhance the maximized operating surplus ϕ_{t+1} in (4) by selecting a different $k_{i,t+1}$. Consequently, by reducing information on the precise capital profile $(k_{j,t+1})_j$ to its aggregate K_{t+1}, an intermediate producer forgoes some profits that could be obtained by adjusting its capital according to future demand conditions.

We can quantify the lost profits from not knowing the detailed capital profile in the model. We focus on firms at the extensive margin of capital adjustment, as those firms experience the largest expected lost profits. The expected operating surplus minus user costs of capital for firms at this margin is equal between an investment spike and a non-spike. When the firm knows the values of (m_{t+1}, w_{t+1}) at time t, it can increase profits by choosing an investment spike when $m_{t+1}/w_{t+1}^{1-\alpha}$ exceeds its steady-state value, opting for inaction otherwise.

We compute the profit increase derived from this strategy under our calibration. The profit gained is small: 0.03% of the steady-state operating surplus. Therefore, the cost incurred by an intermediate producer following a threshold policy in a steady-state economy is negligible. When the cost of obtaining precise information on the capital profile in advance exceeds the small profit gain, the firm will choose to follow the behavioral rule.

Second, the current investment shock ϵ_t influences Φ_t and the expected discounted operating surplus $\mathbb{E}[(\Lambda_{t+1}/\Lambda_t)\mu_{t+1} \mid \epsilon_t]$. An intermediate firm forgoes profits by adopting a static expectation Φ. Under our calibrated parameters, the loss amounts to 0.12% of the firm's operating surplus at the extensive margin. This loss is larger than the cost of uncertainty in ϵ_{t+1} but remains small. In aggregate, households lose 0.84% of dividend revenues. This indicates a connection between our fluctuation mechanism and financial imperfections. The volatility of avalanches could be lower if intermediate firms internalize the response of stochastic discount factors to the avalanche. However, a firm's incentive to do so is not quantitatively significant.

The welfare loss to households stems from coordination failures in investment choices: households seek stability in the extensive margin of aggregate investments, while firms at that margin are strongly incentivized by aggregate investment. Therefore, the aggregate fluctuations in our model are inefficient. A central planner finds it challenging to address this inefficiency, as the investment fluctuations arise from specific realizations of capital and productivity at the firm level. Effective policy intervention to stabilize these fluctuations would require real-time information on detailed capital profiles and necessitate enforcing investment allocations for firms in the extensive margin. Although households face a welfare loss, it is unlikely to be quantitatively significant, as suggested by Lucas (1987) in our representative-household, exogenous-growth setup. The quantification of the welfare loss resulting from investment avalanches in the model of heterogeneous households or endogenous growth is reserved for future research.

4.6 Discussion

To intuitively understand the investment avalanche, we compare it with smoothly adjusted capital and no indivisibility. It is easy to derive the familiar factor price condition in an economy with smoothly adjusted capital (see online appendix),

$$w_t^{1-\alpha}(\Lambda_{t-1}/\Lambda_t - 1 + \delta)^\alpha = \alpha^\alpha (1-\alpha)^{1-\alpha} \frac{\eta - 1}{\eta} \left(\int a_{it}^{\eta-1} di \right)^{\frac{1}{\eta-1}}. \tag{21}$$

Under this condition, the equilibrium marginal cost remains constant, and the optimal capital k_i is directly proportional to K. As a result, any level of K aligns with individual firms' decisions. In a typical constant-returns-to-scale economy, household saving choices determine the level of aggregate capital. Firms' behaviors merely constrain equilibrium factor prices and do not restrict the equilibrium aggregate capital.

When capital is discrete, the aggregate capital adjustment takes place at the firm's extensive margin. Even in this scenario, if infinitely many firms exist, the aggregate capital is indeterminate from the producers' behaviors, and the irrelevance result holds as demonstrated by Thomas (2002). However, if the number of firms is finite, it no longer follows that any level of aggregate capital is consistent with the firms' decisions. Firms' decisions constrain the equilibrium level of aggregate capital. In particular, a socially efficient level of aggregate capital is not generically supported as an equilibrium when it contradicts the firm's decision at the extensive margin. When the firm invests at the extensive margin, it increases aggregate capital. This investment encourages other firms to follow suit due to the complementarity of investment decisions in equilibrium, leading to an avalanche effect. Our analysis showed that firms' behaviors restrict the set of possible equilibria so narrowly that even the least volatile equilibrium path displays significant fluctuations.

Our results are robust when incorporating smoothly adjusting sectors into the model. The analysis above indicates that continuously adjusting firms choose their capital linearly to aggregate capital under equilibrium factor prices satisfying (21), which all firms anticipate according to our behavioral assumption. Consequently, the investment choices of the smooth sector adjust proportionately to the aggregate capital in the lumpy sector.

We can interpret the investment fluctuations in our model as a coordination failure (Cooper and John, 1988): even though representative households desire a smooth path of aggregate investments, the decentralized decision-making of firms diverges from this desired path. The mechanism of coordination failure shares similarities with sunspot models, as the complementarity in investment decisions leads to global indeterminacy. Our model differs from indeterminacy models in that the equilibrium aggregate capital is locally unique, and we focus on the least volatile equilibrium path.

Our model shares with Brock and Durlauf (2001) that multiple equilibria arise from discrete choices. The emergence of multiple equilibria depends on the strength of complementarity in their model, which is analogous to our finding that aggregate fluctuations depend on $\tilde{\theta}$. Our paper differs in that we focus on stochastic fluctuations when n is large but finite. Our model characterizes the distribution of fluctuations around the steady state when the complementarity parameter $\tilde{\theta}$ is near the phase-transition point, incorporating this mechanism within a standard framework of business cycle models.

In standard business cycle models, an investment demand shock is mitigated by a general equilibrium effect that operates through interest rates, as clarified by Khan and Thomas (2008). In our model, the firms' behavioral rule $k^*_{it} = a_{it}^{\eta-1}\Phi K_t$ bypasses this powerful effect. The firms' rule-of-thumb expectation for the constant Φ is grounded in the fact that expected factor prices Φ_t remain constant in the equilibrium of an economy with infinitely many firms. Furthermore, when a firm's procurement of investment takes time after its investment decision, concurrent avalanche shocks at the time of the decision do not directly affect the relevant discount factor, which is determined by the timing of procurement and production.

It is essential to understand how quickly the investment shock and subsequent consumption growth are reflected in the stochastic discount factor faced by firms. This occurs instantaneously in the general equilibrium case of Khan and Thomas (2008). Authors such as Auclert, Rognlie, and Straub (2020), Koby and Wolf (2020), Winberry (2021), and Zwick and Mahon (2017) indicate sluggish responses of investment to interest rates. Christiano, Motto, and Rostagno (2014) emphasizes the significant role of credit spreads in business cycles. Angeletos (2018) argues for the absence of common knowledge among economic agents, leading to dampened general equilibrium effects. Our paper extends this discussion by highlighting that shocks emerge from the interaction of lumpy investments against the backdrop of imperfect financial markets.

5 Conclusion

"Animal spirits" haunt discussions of the business cycle. In the regular ups and downs of business, it appears that aggregate investment demand is driven by the whims of firms. However, a solid mechanism generating animal spirits has yet to be identified. This paper presents a model that explains how shocks to aggregate investment demand arise. We assume that a firm's capital is indivisible, resulting in lumpy investment at the firm level. Additionally, we consider a monopolistically competitive economy where an increase in aggregate demand encourages firms to invest. In this setup, the lumpy investment of one firm prompts another firm's lumpy investment, triggering an investment avalanche. The extent to which the investment avalanche continues depends on the capital profile and the idiosyncratic productivity of the firms. Thus, as the capital and productivity profiles evolve, the size of the avalanche fluctuates. This leads to variations in the size of the investment avalanche, which we refer to as investment demand shocks.

The paper analyzes the fluctuations of the investment avalanche both analytically and numerically. Our analysis indicates that the investment demand shocks generated by the avalanche are quantitatively significant. Under time-to-build conditions, the investment demand shocks can display autocorrelation. Therefore, the investment avalanche offers a microfoundation for the animal spirits that drive investment demands in business cycles.

We incorporate the mechanism of investment avalanches into an otherwise standard New Keynesian business cycle model. With the rigidity of real wages, the model allows for analytic solutions and interpretation of the propagation mechanism of micro-founded investment demand shocks. Analyzing the impulse-response dynamics of an investment demand shock reveals that this shock can generate a positive comovement of inflation, consumption, investment, and output in the business cycle frequency under reasonable parameter alignment.

Appendix

A Generalization to decreasing returns to scale

This section shows that the recursive equilibrium defined in Section 2 naturally generalizes to the decreasing returns to scale case, which is used in Section 4. The production function of intermediate producer j is set as $y_{jt}^m = a_{jt}(k_{jt}^{\alpha} l_{jt}^{1-\alpha})^{\theta}$, where $\theta \leq 1$. Other fundamental setups remain the same as Section 2.

A.1 Production

Facing the demand from wholesales $y_{jt}^m = \left(p_{jt}^m / P_t^m\right)^{-\eta} Y_t$, intermediate firm j chooses labor demand as $l_{jt} = (1-1/\eta)(1-\alpha)\theta(m_t/w_t)(y_{jt}^m)^{1-1/\eta} Y_t^{1/\eta}$. Aggregating across j, we obtain aggregate goods supply and labor demand functions:

$$Y_t = ((1-1/\eta)(1-\alpha)\theta m_t/w_t)^{\frac{(1-\alpha)\theta}{1-(1-\alpha)\theta}} K_t^{\frac{\alpha\theta}{1-(1-\alpha)\theta}}, \tag{22}$$

$$L_t = \sum_{j=1}^{n} l_{jt}/n = (1-1/\eta)(1-\alpha)\theta(m_t/w_t)Y_t, \tag{23}$$

where $K_t = \left(\sum_{j=1}^n (a_{jt}^{1/(\alpha\theta)} k_{jt})^{\rho}/n\right)^{1/\rho}$ with an abuse of notation $\rho := \frac{(1-1/\eta)\alpha\theta}{1-(1-1/\eta)(1-\alpha)\theta}$. This newly defined ρ nests the one in Section 2 as a special case of $\theta = 1$. Note that $0 < \rho < 1$ is satisfied since $\eta > 1$, $\theta \leq 1$, and $0 < \alpha < 1$. The operating surplus is written as:

$$\mu_t(a_{jt}, k_{jt}) = \kappa(a_{jt}^{1/(\alpha\theta)} k_{jt})^{\rho} \left(m_t/w_t^{(1-\alpha)\theta}\right)^{\frac{1}{1-(1-\alpha)\theta}} K_t^{\frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-(1-1/\eta)(1-\alpha)\theta}} \tag{24}$$

where $\kappa := (1-(1-1/\eta)(1-\alpha)\theta)\left((1-1/\eta)(1-\alpha)\theta\right)^{\frac{(1-\alpha)\theta}{1-(1-\alpha)\theta}}$. Aggregating (24) yields an expression for aggregate operating surplus:

$$\sum_{j=1}^{n} \mu_t(a_{jt}, k_{jt})/n = \kappa \left(m_t/w_t^{(1-\alpha)\theta}\right)^{\frac{1}{1-(1-\alpha)\theta}} K_t^{\frac{\alpha\theta}{1-(1-\alpha)\theta}}. \tag{25}$$

A.2 Lumpy investment

Using the indifference condition as in Section 2, the threshold k^* for investment spikes is obtained as

$$k^*_{j,t+1} = \tilde{a}_{j,t+1}\Phi_t K^{\tilde{\theta}}_{t+1}, \tag{26}$$

where $\tilde{a}_{j,t} := a_{j,t}^{\frac{1-1/\eta}{1-\theta+\theta/\eta}}$, $\tilde{\theta} := \frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}$, and Φ_t summarizes the expected factor prices

$$\Phi_t := \left(\kappa\frac{\lambda^\rho - 1}{\lambda - 1}\mathbb{E}_t\left[\Lambda_{t+1}\left(m_{t+1}/w_{t+1}^{(1-\alpha)\theta}\right)^{\frac{1}{1-(1-\alpha)\theta}}\right]\mathbb{E}_t\left[\Lambda_t - \Lambda_{t+1}(1-\delta)\right]^{-1}\right)^{\frac{1}{1-\rho}}.$$

Aggregate capital is written by using F_t as $K_t = \left(\sum_{j=1}^n (a_{jt}^{1/(\alpha\theta)} k_{jt})^\rho/n\right)^{1/\rho} = \mathbb{E}^{F_t}\left[(a_{jt}^{1/(\alpha\theta)}\lambda^{s_{jt}}k^*_{jt})^\rho\right]^{1/\rho}$. Substituting $k^*_{j,t+1}$ into this expression for K_{t+1} yields an equilibrium condition for factor prices Φ_t:

$$1 = \mathbb{E}^{F_{t+1}}\left[\tilde{a}\lambda^{\rho s}\right]^{1/\rho}\Phi_t K_{t+1}^{\tilde{\theta}-1}. \tag{27}$$

A.3 Recursive equilibrium when $n \to \infty$

Households behavior, monetary policy, and market-clearing conditions are unchanged from Section 2. Under Assumption 1, $F_t(a,s)$ stays at the stationary distribution F. Writing $B := \mathbb{E}^F[\tilde{a}\lambda^{\rho s}]^{-1/\rho}$,[6] (27) implies $\Phi_t = BK_{t+1}^{1-\tilde{\theta}}$, leading to:

$$u'(C_t) = \beta\mathbb{E}_t\left[u'(C_{t+1})\left(\frac{\kappa}{B^{1-\rho}}\frac{\lambda^\rho - 1}{\lambda - 1}\left(\frac{w_{t+1}^{(1-\alpha)\theta}}{m_{t+1}}\right)^{\frac{-1}{1-(1-\alpha)\theta}} K_{t+1}^{-\frac{1-\theta}{1-(1-\alpha)\theta}} + 1 - \delta\right)\right].$$

Under Assumption 1, the law of motion for aggregate capital holds as (14). This is because $\Phi_t = BK_{t+1}^{1-\tilde{\theta}}$ with (26) implies $k^*_{it} = \tilde{a}_{it}BK_t$. Noting B equals Φ, the threshold policy in a general equilibrium remains the same as in Section 2. Hence, the derivation of (14) in Section 2 holds here.

In the limit of n, the recursive equilibrium of $(Y_t, K_{t+1}, X_t, L_t, N_t, C_t, w_t, m_t, i_t, \pi_t)$ is determined by (1,9,10,11,12,13,14,22,23,27) under Assumption 1. We write $K_{t+1} = \Xi(K_t)$ for a mapping of aggregate capital that the recursive equilibrium determines.

[6] B has the same value as Φ in Section 2, but we use a different notation because the steady-state value of Φ_t differs from Φ for $\theta < 1$.

B Proofs

B.1 Proposition 1

The threshold rule (26) converges to $k^*_{i,t} = \tilde{a}_{i,t}\Phi_{t-1}K_t^{\tilde{\theta}}$ when $n \to \infty$ where $\tilde{a} = a^{(1-1/\eta)/(1-\theta+\theta/\eta)}$ and $\tilde{\theta} = (\alpha\theta/\eta)/(1-(1-\alpha)\theta)/(1-\theta+\theta/\eta)$. Since firm i's capital satisfies $k_{i,t} = \lambda^{s_{i,t}}k^*_{i,t}$, we obtain

$$K_t = \mathbb{E}^F\left[(a_{it}^{1/(\alpha\theta)}\lambda^{s_{it}}k^*_{it})^\rho\right]^{1/\rho} = \mathbb{E}^F\left[(a_{it}^{1/(\alpha\theta)}\lambda^{s_{it}}\tilde{a}_{i,t})^\rho\right]^{1/\rho}\Phi_{t-1}K_t^{\tilde{\theta}}$$
$$= \mathbb{E}^F\left[\tilde{a}_{it}\lambda^{\rho s_{it}}\right]^{1/\rho}\Phi_{t-1}K_t^{\tilde{\theta}}. \tag{28}$$

Consider a perturbation $\nu(h)$ from the stationary threshold $s^*(h)$. A perturbed aggregate capital evaluated at the stationary equilibrium satisfies the following equation.

$$K^\rho_{t+1} = \sum_{h=1}^{H^2}\omega(h)\left[\int_0^{s^*(h)+\nu(h)}(a^{1/(\alpha\theta)}(h_1)\lambda(1-\delta)\lambda^{s_{it}}\tilde{a}(h_0)BK_t^{\tilde{\theta}})^\rho ds\right.$$
$$\left.+\int_{s^*(h)+\nu(h)}^1(a^{1/(\alpha\theta)}(h_1)(1-\delta)\lambda^{s_{it}}\tilde{a}(h_0)BK_t^{\tilde{\theta}})^\rho ds\right]$$

We fix h so that $\nu(h)>0$ and $\nu(h')=0$ for $h'\neq h$. Then, we have $\partial\rho\log K_{t+1}/\partial\nu(h) = \omega(h)(\lambda^\rho-1)\left(a^{1/(\alpha\theta)}(h_1)(1-\delta)\lambda^{s^*(h)+\nu(h)}\tilde{a}(h_0)BK_t^{\tilde{\theta}}\right)^\rho/K^\rho_{t+1}$. This expression is evaluated at $\nu(h)=0$ by using $s^*(h) = (-\log(1-\delta)+\Delta\log\tilde{a}(h))/\log\lambda$ and (28) as

$$\left.\frac{\partial(\rho\log K_{t+1})}{\partial\nu(h)}\right|_{\nu=0} = \frac{\omega(h)(\lambda^\rho-1)\left(a^{1/(\alpha\theta)}(h_1)(1-\delta)\lambda^{s^*(h)}\tilde{a}(h_0)BK_t^{\tilde{\theta}}\right)^\rho}{K^\rho_{t+1}\big|_{\nu=0}}$$
$$= \omega(h)(\lambda^\rho-1)\left(a^{1/(\alpha\theta)}(h_1)\tilde{a}(h_1)(1-\delta)\lambda^{-\frac{\log(1-\delta)}{\log\lambda}}\right)^\rho\frac{1}{\mathbb{E}^F[\tilde{a}\lambda^{\rho s}]}$$
$$= \omega(h)\rho(\log\lambda)\frac{\tilde{a}(h_1)}{\mathbb{E}^F[\tilde{a}]},$$

where the last equality used Assumption 1 that s is uniformly distributed conditional on every a.

Under the uniform distribution of s_{it}, an increase in $\log K_{t+1}$ increases the threshold by $\tilde{\theta}d\log K_{t+1}/\log\lambda$. Hence, we obtain

$$\vartheta(h) := \lim_{\nu(h)\to 0}\frac{\nu'}{\omega(h)\nu(h)} = \frac{d\nu'/d\nu(h)|_{\nu(h)=0}}{\omega(h)} = \frac{\tilde{\theta}/\log\lambda}{\omega(h)}\left.\frac{\partial\log K_{t+1}}{\partial\nu}\right|_{\nu(h)=0} = \frac{\tilde{\theta}\tilde{a}(h_1)}{\mathbb{E}^F[\tilde{a}]}.$$

When we perturb ν measure of firms unconditional on h, we obtain the degree of complementarity as the average of $\vartheta(h)$,

$$\vartheta = \sum_h \omega(h)\vartheta(h) = \tilde{\theta} = \frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}.$$

We note that the right-hand side function is increasing in θ for $\theta \leq 1$ and converges to 1 as $\theta \to 1$.

B.2 Proofs for statements in Section 4.3

B.2.1 Distributions of ϵ_0^n and n_δ

In step 1 of the equilibrium selection algorithm, capital is depreciated by δ. In step 2, productivity a_{it} is updated. In the first round of step 3, firms with $s_{it} \leq s_{it}^* := -(\log(1-\delta) - \Delta \log \tilde{a}_{it})/\log\lambda$ invest.

The net increase of $\log K_t^n$ due to their investments is denoted by ϵ_0^n. Hence,

$$\begin{aligned}
\epsilon_0^n &= \frac{1}{\rho}\left(\log\left(\sum_{i:s_{it}\leq s_{it}^*} \frac{(a_{i,t+1}^{1/(\alpha\theta)}\lambda(1-\delta)k_{it})^\rho}{n} + \sum_{i:s_{it}>s_{it}^*} \frac{(a_{i,t+1}^{1/(\alpha\theta)}(1-\delta)k_{it})^\rho}{n}\right) - \log\sum_i \frac{(a_{it}^{1/(\alpha\theta)}k_{it})^\rho}{n}\right) \\
&= \frac{1}{\rho}\left(\log\left((\lambda^\rho - 1)\sum_{i:s_{it}\leq s_{it}^*} \frac{(a_{i,t+1}^{1/(\alpha\theta)}\lambda^{s_{it}})^\rho}{n} + \sum_i \frac{(a_{i,t+1}^{1/(\alpha\theta)}\lambda^{s_{it}})^\rho}{n}\right) - \log\sum_i \frac{(a_{it}^{1/(\alpha\theta)}\lambda^{s_{it}})^\rho}{n}\right) \\
&\quad + \log(1-\delta).
\end{aligned}$$

As $n \to \infty$, ϵ_0^n degenerates to zero, and $\sqrt{n}\epsilon_0^n$ converges to a non-degenerate random variable with a finite variance by the central limit theorem.

We divide firms into H^2 groups according to their profile $(a_{it}, a_{i,t+1})$ as in Section 2. Let $n(h)$ denote the number of firms in group h. Then, the number of firms in h that invest in step 3, denoted by $n_\delta(h)$, follows a binomial distribution with population $n(h)$ and probability $s^*(h)$. The total number of firms investing in this step is determined by the sum of the binomials over h, $n_\delta = \sum_h n_\delta(h)$.

B.2.2 Distribution of z_0^n

Suppose $\epsilon_0^n > 0$. Step 4 of the equilibrium selection algorithm is repeated. z_0^n denotes the number of firms in $[s_{it}^*, s_{it}^* + \tilde{\theta}\epsilon_0^n/\log\lambda)$. If $z_0^n = 0$, the algorithm stops, and an equilibrium capital in t is determined. If $z_0^n > 0$, the algorithm continues.

Let $z_0^n(h)$ denote the number of firms that belong to group h and are located in $[s^*(h), s^*(h)+\tilde{\theta}\epsilon_0^n/\log\lambda)$. The firms that do not invest in step 3 and belong to group h are in $[s^*(h), 1)$ uniformly. Hence, $z_0^n(h)$ follows a binomial distribution with population $n(h) - n_\delta(h)$ and probability $\tilde{\theta}\epsilon_0^n/((1-s^*(h))\log\lambda)$. We note that the mean of $n_\delta(h)/n(h)$ is $s^*(h)$. Hence, the mean of $z_0^n(h)$ is $n(h)\tilde{\theta}\epsilon_0^n/\log\lambda$. Also, the mean of $z_0^n = \sum_h z_0^n(h)$ is $n\tilde{\theta}\epsilon_0^n/\log\lambda$.

If $\epsilon_0^n < 0$, step 4 of the equilibrium selection algorithm searches for firms in $(s_{it}^* + \tilde{\theta}\epsilon_0^n/\log\lambda, s_{it}^*]$ that "retract" the investment decision they made in step 3. We denote the number of retracted firms by a negative of z_0^n.

B.2.3 Mean of z_u^n

In a model with n firms, the average capital level satisfies $(K^n)^\rho = \sum_{i=1}^n (a_i^{1/(\alpha\theta)}k_i)^\rho/n$. If k_i^ρ incerases to $(\lambda k_i)^\rho$, $(K^n)^\rho$ increases by $(\lambda^\rho - 1)(a_i^{1/(\alpha\theta)}k_i)^\rho/n$. Hence, firm i's lumpy investment increases $\log(K^n)$ by

$$\Delta\log(K^n) := \frac{\lambda^\rho - 1}{\rho}\frac{(a_i^{1/(\alpha\theta)}k_i)^\rho}{\sum_{j=1}^n (a_j^{1/(\alpha\theta)}k_j)^\rho} + o\left(\frac{(a_i^{1/(\alpha\theta)}k_i)^\rho}{\sum_{j=1}^n (a_j^{1/(\alpha\theta)}k_j)^\rho}\right).$$

Using normalized capital $s_i = (\log k_i - \log k_i^*)/\log\lambda$, we have $k_i = \lambda^{s_i}k_i^*$. We consider a stationary equilibrium of a continuum model where s_i is distributed uniformly over $[0,1)$ and independent of a_i. We also use the threshold rule for a stationary equilibrium of a continuum model: $k_j^* = \tilde{a}_j B(K^n)^{\tilde{\theta}}$. Then, i's investment decreases s_j for $j \neq i$ by $\Delta s_j = -\tilde{\theta}\Delta\log(K^n)/\log\lambda$. Since s_j is uniformly distributed, the number of firms (in the group h) that hit the threshold because of this decrease in s_j follows a binomial distribution with population $n - n_\delta h - \sum_{\tau=0}^{u-1} z_\tau^n(h)$ and probability $\Delta s_j/(1-s^*(h))$.

We use $(a_i^{1/(\alpha\theta)}k_i)^\rho = (a_i^{1/(\alpha\theta)}\lambda^{s_i}\tilde{a}_i B(K^n)^{\tilde{\theta}})^\rho = \tilde{a}_i(\lambda^{s_i}B(K^n)^{\tilde{\theta}})^\rho$. Thus, for a given sequence of $(z_\tau^n(h))_{\tau=0}^{u-1}$, the mean of $z_u^n = \sum_h z_u^n(h)$ is

$$\operatorname*{plim}_{n\to\infty}\sum_h\sum_{i\in\mathcal{Z}_{u-1}(h)}\frac{\tilde{\theta}\Delta\log(K^n)}{(1-s^*(h))\log\lambda}\left(n - n_\delta(h) - \sum_{\tau=0}^{u-1} z_\tau^n(h)\right) = \operatorname*{plim}_{n\to\infty}\sum_{i\in\mathcal{Z}_{u-1}}\frac{\lambda^\rho - 1}{\rho\log\lambda}\frac{\tilde{\theta}\tilde{a}_i\lambda^{\rho s_i}}{\sum_{j=1}^n \tilde{a}_j\lambda^{\rho s_j}/n}.$$

By the law of large numbers, $\sum_{j=1}^n \tilde{a}_j\lambda^{\rho s_j}/n$ converges in probability as $n \to \infty$ to $\mathbb{E}^F[\tilde{a}](\lambda^\rho - 1)/(\rho\log\lambda)$. Also, note that $s_i \to 0$ as $n \to \infty$ for $i \in \mathcal{Z}_{u-1}$. Hence, the mean of z_u^n conditional on $\mathcal{Z}_{u-1}$ is asymptotically equal to $\tilde{\theta}\sum_{i\in\mathcal{Z}_{u-1}}\tilde{a}_i/\mathbb{E}^F[\tilde{a}]$.

In particular, the mean of z_u^n conditional on that $\mathcal{Z}_{u-1}$ contains a single firm in group h converges as $n \to \infty$ to $\vartheta(h) = \tilde{\theta}\tilde{a}(h_1)/\mathbb{E}[\tilde{a}]$, which is the degree of complementarity we obtained in Proposition 1 in the model with a continuum of firms. Thus, the mean number of firms induced to invest due to an investing firm (unconditional on h), ϑ, converges in probability to $\sum_h \omega(h)\vartheta(h) = \tilde{\theta}$ as $n \to \infty$. In this way, the degree of complementarity ϑ defined in the continuum model characterizes the avalanche effect of investments in a model of a finite number of firms.

Since Poisson distribution is an asymptotic distribution of a binomial distribution with a finite mean, z_u converges in law to a Poisson distribution with mean ϑz_{u-1}. Furthermore, since a Poisson distribution is infinitely divisible and z_{u-1} is an integer, the asymptotic distribution of z_u is equivalent to a z_{u-1}-times convolution of a Poisson distribution with mean ϑ.

The proof of Proposition 6 of Nirei and Scheinkman (2024) applies to our L^n and L because ours is a special case of the proposition when the density of state (s_i) is constant. Their Propositions 8a, b, and d also hold in our case because L conditional on z_0 is the cumulative sum of a Poisson branching process.

C Time to build

C.1 Time to build with equally spread investments and jump in capital

Let $s_{it} := k_{i,t+J} - (1-\delta)k_{i,t+J-1}$ denote the increment of capital that materializes in J periods later. The investment s_{it} is spent over J periods from t to $t+J-1$ with weight for each period ζ_τ, $\tau = 0, 1, \ldots, J-1$. Thus, firm i's total investments x_{it} in period t is a weighted sum of past s_{it} as $x_{it} = \sum_{\tau=0}^{J-1} \zeta_\tau s_{i,t-\tau}$. The real value of intermediate firm i is $\mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_t(\mu_t(a_{it}, k_{it}) - x_{it})$.

Let Λ_t^J denote a J-period rolling average as $\Lambda_t^J := \sum_{\tau=0}^{J-1} \zeta_\tau \Lambda_{t+\tau}$. Then, the terms relevant to the choice of k_{t+J} is:

$$\mathbb{E}_t \Lambda_{t+J}\mu_{t+J}(a_{i,t+J}, k_{i,t+J}) + (1-\delta)\Lambda_{t+1}^J k_{t+J} - \Lambda_t^J k_{t+J}.$$

The firm's threshold for lumpy investment is solved as in the previous section. First, the indifference condition between an investment spike in $t+J$ or $t+J+1$ yields

$$k^*_{i,t+J} = \tilde{a}_{i,t+J}\Phi^J_t K^{\tilde{\theta}}_{t+J}$$

where Φ^J_t summarizes the expected factor prices

$$\Phi^J_t := \left(\kappa\frac{\lambda^\rho - 1}{\lambda - 1}\mathbb{E}_t\left[\Lambda_{t+J}\left(m_{t+J}/w^{(1-\alpha)\theta}_{t+J}\right)^{\frac{1}{1-(1-\alpha)\theta}}\right]\mathbb{E}_t\left[\Lambda^J_t - \Lambda^J_{t+1}(1-\delta)\right]^{-1}\right)^{\frac{1}{1-\rho}}. \tag{29}$$

The steady-state values are affected by the change in stochastic discount factors in (29). At the steady state, $\Lambda^J_t/\Lambda_{t+J}$ becomes $\sum_{\tau=0}^{J-1}\zeta_\tau\Lambda_{t+\tau}/\Lambda_{t+J} = \sum_{\tau=0}^{J-1}\zeta_\tau\beta^{\tau-J} =: (\beta_J)^{-1}$. Also, at the steady state, $\Lambda^J_t/\Lambda^J_{t+1} = \beta^{-1}$. The equation at the steady state yields:

$$\Phi^J := \left(\kappa\frac{\lambda^\rho - 1}{\lambda - 1}\frac{\beta_J}{\beta}w^{\frac{-(1-\alpha)\theta}{1-(1-\alpha)\theta}}\left[\frac{1}{\beta} - (1-\delta)\right]^{-1}\right)^{\frac{1}{1-\rho}}. \tag{30}$$

In Section 3.4, we used a specification in which capital gradually builds up over the time-to-build periods. The extension of the model to the gradual capital build-up is straightforward. We present the extension in the next section.

C.2 Gradual capital build-up

In this section, we specify that a lumpy investment takes J periods to complete and capital increases gradually during the J periods. Suppose that a firm decides a lumpy investment in period t. Then, the firm commits to a series of investment purchases $(s_t, s_{t+1}, \ldots, s_{t+J-1})$. Capital develops as

$$k_{t+j} = (1-\delta)^j k_t + (1-\delta)^{j-1}s_t + (1-\delta)^{j-2}s_{t+1} + \cdots + s_{t+j-1}.$$

We impose that the cumulated capital increase corresponds to the lumpiness parameter λ. Thus,

$$k_{t+J} = (1-\delta)^J k_t + (1-\delta)^{J-1}s_t + (1-\delta)^{J-2}s_{t+1} + \cdots + s_{t+J-1} = \lambda(1-\delta)k_t.$$

Moreover, we impose that j-th period investment s_{t+j} is proportional to k_t:

$$s_{t+j} = \zeta_j k_t.$$

Hence $\zeta_j \geq 0$, $j = 0, 1, \ldots, J-1$, satisfies a condition

$$(1-\delta)^J + (1-\delta)^{J-1}\zeta_0 + (1-\delta)^{J-2}\zeta_1 + \cdots + \zeta_{J-1} = \lambda(1-\delta).$$

There always exists a series $(\zeta_j)_{j=0}^{J-1}$ which satisfies the condition. In particular, when $J = 1$, the model is reduced to the simplest case with $\zeta_0 = (\lambda - 1)(1-\delta)$.

Substituting $s_{t+j} = \zeta_j k_t$, we have

$$k_{t+j} = \xi_j k_t \quad \text{for } j = 1, 2, \ldots, J,$$

where $\xi_j := (1-\delta)^j + (1-\delta)^{j-1}\zeta_0 + (1-\delta)^{j-2}\zeta_1 + \cdots + \zeta_{j-1}$.

At the threshold k^* of investment decision, a firm is indifferent between starting investment in t and $t+1$. The two paths differ in capital between $t+1$ and $t+J$ and follow the same path after $t+J+1$. Using (24), firm i's operating surplus is

$$\mu_t(a_{it}, k_{it}) = k_{it}^{\rho} B(a_{it}, w_t, m_t, K_t) \tag{31}$$

where

$$B(a_{it}, w_t, m_t, K_t) := a_{it}^{\rho/(\alpha\theta)} \kappa \left(m_t / w_t^{(1-\alpha)\theta}\right)^{\frac{1}{1-(1-\alpha)\theta}} K_t^{\frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-(1-1/\eta)(1-\alpha)\theta}}$$

and $\kappa := (1-(1-1/\eta)(1-\alpha)\theta)\left((1-1/\eta)(1-\alpha)\theta\right)^{\frac{(1-\alpha)\theta}{1-(1-\alpha)\theta}}$.

If we write the difference in operating surplus in t between the two paths as $\Delta\mu_t$, we have

$$\Delta\mu_{t+j} = B(a_{i,t+j}, w_{t+j}, m_{t+j}, K_{t+j})(\xi_j^{\rho} - (\xi_{j-1}(1-\delta))^{\rho})k_{i,t+j}^{\rho} \quad \text{for } j = 1, \ldots, J.$$

The difference in investment cost in $t+j$ is $\zeta_j k_t - \zeta_{j-1}(1-\delta)k_t$ for $j = 0, \ldots, J-1$ where $\zeta_{-1} = 0$ by convention. Let k^* denote the capital level in $t+1$ at which investment in t is indifferent. Thus, in t, a firm is indifferent in investing if $k_t = k^*/(1-\delta)$. Hence, $k^*/(1-\delta)$ satisfies

$$\begin{aligned}
&\mathbb{E}_t\left[\sum_{j=1}^{J} \Lambda_{t+j} B(a_{i,t+j}, w_{t+j}, m_{t+j}, K_{t+j})(\xi_j^{\rho} - (\xi_{j-1}(1-\delta))^{\rho})\left(\frac{k^*}{1-\delta}\right)^{\rho}\right] \\
&= \mathbb{E}_t\left[\sum_{j=0}^{J} \Lambda_{t+j}(\zeta_j - \zeta_{j-1}(1-\delta))\right]\frac{k^*}{1-\delta}
\end{aligned}$$

where $\zeta_{-1} = \zeta_J = 0$ by convention. The optimal threshold is solved as

$$\frac{k^*}{1-\delta} = \left(\frac{\mathbb{E}_t\left[\sum_{j=1}^{J} \Lambda_{t+j} B(a_{i,t+j}, w_{t+j}, m_{t+j}, K_{t+j})(\xi_j^{\rho} - (\xi_{j-1}(1-\delta))^{\rho})\right]}{\mathbb{E}_t\left[\sum_{j=0}^{J} \Lambda_{t+j}(\zeta_j - \zeta_{j-1}(1-\delta))\right]}\right)^{1/(1-\rho)}$$

$$= \mathbb{E}_t \left[\sum_{j=1}^{J} \tilde{\Lambda}_{t+j} B(a_{i,t+j}, w_{t+j}, m_{t+j}, K_{t+j}) \right]^{1/(1-\rho)}$$

where

$$\tilde{\Lambda}_{t+j} := \frac{\Lambda_{t+j}(\xi_j^\rho - (\xi_{j-1}(1-\delta))^\rho)}{\mathbb{E}_t \left[\sum_{j=0}^{J} \Lambda_{t+j}(\zeta_j - \zeta_{j-1}(1-\delta)) \right]}.$$

We impose $\theta = 1$ and $a_{i,t}$ i.i.d. across i and t. Let $\tilde{\kappa} = \kappa \mathbb{E}_t[(a_{t+1}/a_t)^{\rho/\alpha}]$. Then,

$$\begin{aligned}
\left(\frac{k^*}{1-\delta}\right)^{1-\rho} &= \mathbb{E}_t \left[\sum_{j=1}^{J} \tilde{\Lambda}_{t+j} B(a_{i,t+j}, w_{t+j}, m_{t+j}, K_{t+j}) \right] = \mathbb{E}_t \left[\sum_{j=1}^{J} \tilde{\Lambda}_{t+j} a_{i,t+j}^{\rho/\alpha} \kappa (m_{t+j}/w_{t+j}^{1-\alpha})^{1/\alpha} K_{t+j}^{1-\rho} \right] \\
&= a_{i,t}^{\rho/\alpha} \mathbb{E}_t \left[\sum_{j=1}^{J} \tilde{\Lambda}_{t+j} \tilde{\kappa} (m_{t+j}/w_{t+j}^{1-\alpha})^{1/\alpha} K_{t+j}^{1-\rho} \right]
\end{aligned}$$

By definition of s_{it}, we have $k_{i,t+1} = \lambda^{s_{i,t+1}}(1-\delta)k_{i,t}^*$. Recall the assumption that $a_{i,t+J}$ is known in t. Thus,

$$\begin{aligned}
K_{t+1}^\rho &= \int (a_{i,t+1}^{1/\alpha} k_{i,t+1})^\rho di \\
&= (1-\delta)^\rho \int \left(a_{i,t+1}^{\eta-1+1/\alpha} \lambda^{s_{i,t+1}} \right)^\rho di \mathbb{E}_t \left[\sum_{j=1}^{J} \tilde{\Lambda}_{t+j} \tilde{\kappa} (m_{t+j}/w_{t+j}^{1-\alpha})^{1/\alpha} K_{t+j}^{1-\rho} \right]^{\rho/(1-\rho)}
\end{aligned}$$

Hence, under the assumption that (a, s) follows the stationary distribution F,

$$k^* = a_{i,t}^{\eta-1} B K_{t+1}$$

where $B = \mathbb{E}^F[a^{\eta-1}\lambda^{\rho s}]^{-1/\rho}$. Thus, we recover the formula for the investment threshold.

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